

# Force Control in Multi-degree-of-freedom Flexible Systems - Sensorless Technique

Islam S. M. Khalil, A. T. Naskali and Asif Sabanovic

**Abstract**—This paper presents a novel sensorless force control algorithm for Multi-degree-of-freedom flexible systems which enables controlling the interaction forces with the environment without using force sensors. The coupled nature of flexible system dynamics makes it possible to estimate externally applied forces or torques that arise due to system's interaction with the environment.

Disturbance and flexibility are simultaneously utilized to estimate system parameters, dynamics and externally applied forces or torques. The interaction torque estimate is then used to accomplish sensorless torque control assignment. This paper attempts to keep the flexible plant free from any measurement while performing a torque control assignment. However, actuator's parameters and variables are assumed to be available.

## I. INTRODUCTION

Grasping, assembly, machining and many other applications necessitate controlling task space forces which in turn necessitates the utilization of force sensors to provide the control system with the necessary force feedback information. However, force sensors extremely increase the initial cost of the system along with the sophisticated hardware they add in terms of additional electronic setups and wirings. Therefore, interest of sensorless force control techniques is ever-growing due to the simplicity they impose to the system, the lower cost they help to achieve and avoidance of sensor noise that causes degradation of the control performance.

The narrow bandwidth, signal noise and colocation problems were avoided in [1]-[2], where force sensor is replaced with sensorless control system based on a reaction torque observer. Force error observer was proposed in [3] and enabled to realize the force control without force sensor by only using the velocity information. Similarly, the reaction force is calculated from the estimated disturbance torque and the robot inverse dynamics to realize the sensorless force control [4]-[5].

Unlike the previous disturbance observer based techniques [6], vision is used by many researchers to extract force information out the captured images taken from objects subjected to externally applied forces. Interaction forces between the end effector and flexible objects were estimated using visual feedback by mapping the flexible object deformation to the end effector forces [7].

In addition, micro manipulation used in variety of applications such as micro surgery and cells manipulations that necessitates using micro force sensors that in turn increases

the process sophistication. Pushing was utilized by some authors in order to manipulate objects in micro scale by using piezoresistive micro cantilever with lightly doped strain gages that is utilized as force sensor [13].

This paper attempts to achieve sensorless control tasks using multi-degree-of-freedom flexible systems by taking two measurement from the actuator while keeping the flexible plant free from any attached sensors. Disturbance and flexibility are combined to formulate force estimation algorithm based on the fact that disturbance is a rich piece of information that contains system parameters, dynamics and interaction forces in a coupled fashion. In addition, disturbances can be estimated from system's boundaries at which actuator is located. Therefore, actuator's variables are used to estimate this coupled reflected disturbance from the flexible plant.

Despite of the coupled nature of the disturbance signal, plant flexibility [8][9] can be used as an efficient tool to decouple each piece of information out of the total disturbance. Strictly speaking, flexible systems have different behavior along their frequency range, rigid behavior can be observed at their low frequency range. While flexible behavior is expected if any of their flexible modes is excited. Moreover, at certain frequencies some lumped masses of the flexible multi-degree-of-freedom system are not moving at all. Therefore, system flexibility makes it possible to minimize number of coordinates describing the motion of the system, which in turn implies the capability of eliminating number of unknown coordinates. Consequently, the coupled disturbance signal is simplified as many unknown coordinates are dropped at these particular frequencies, namely natural frequencies of the system.

In this paper, Actuator variables are measured and used to estimate disturbances. Then reflected load from the plant to the actuator is decoupled out of this disturbance. A low pass or notch filter is then designed to excite certain modes of the flexible plant to estimate its uniform parameters. Flexible motion of the system is then observed by a chain of recursive observer that depends on the estimated reflected load along with the estimated parameters. Eventually, the previous estimated dynamics are used to extract the externally applied forces on the plant from the disturbance signal.

This paper is organized as follows, velocity and disturbances are estimated then reflected load from the plant is decoupled In section II. Plant dynamics and externally applied torques due to system interaction with the environment are included in section III. Eventually, section IV includes the experimental results along with conclusions and final

Islam S. M. Khalil, A. T. Naskali and Asif Sabanovic are with Faculty of Engineering and natural Sciences, Sabanci University, 34956 Orhanli, Tuzla, Istanbul, Turkey. kahalil@su.sabanciuniv.edu teoman@su.sabanciuniv.edu, asif@sabanciuniv.edu

remarks.

## II. SENSORLESS ESTIMATION ALGORITHM

The following estimation process is based on two measurement from the actuator. Namely, actuator current and velocity, that are required to be measured along with the knowledge of actuator's nominal parameters. The process can be implemented on both distributed and lumped flexible systems, this paper is concerned with the later however the next subsection presents a sensorless method to control forces at the end of a flexible cantilever beam that can be used in micromanipulation processes.

### A. Distributed Flexible Systems

Flexible beam is modeled using the following partial differential equation

$$EIy_{xxxx} + cy_t + \rho Ay_{tt} = f(x,t) \quad (1)$$

where  $E$ ,  $I$ ,  $\rho$ ,  $A$  and  $c$  are modulus of elasticity, Inertia, density, cross section area and damping coefficient of the beam respectively.  $f(x,t)$  is the external applied force while  $y(x,t)$  is the beam's lateral displacement. The flexible beam is subjected to bending moment that can be expressed as

$$M(x,t) = EIy_{xx} \quad (2)$$

therefore  $M(x,t)$  can be shown to be

$$M(x,t) = \frac{1}{2}[f(x,t) - cy_t - \rho Ay_{tt}]x^2 + c_1x + c_2 \quad (3)$$

where  $c_1$  and  $c_2$  are integration constants that depends on the beam's boundary conditions, this bending moment is equivalent to a twisting torque on the actuator. Therefore, the actuator's mechanical equation of motion is

$$J_m \ddot{\theta}_m + B \dot{\theta}_m = i_m k_t + M(x,t) \quad (4)$$

the previous equation along with (3) indicate that beams dynamics in the motion level  $y_t$ , in the acceleration level  $y_{tt}$ , beam's damping coefficient  $c$  and externally applied force  $f(x,t)$  are reflected on the actuator side through the coupled term  $M(x,t)$ . Considering the variation in the actuator's inertia  $J_m$  and torque constant  $k_t$ , 4 can be rewritten as

$$(J_{mo} + \Delta J_m) \ddot{\theta}_m + B \dot{\theta}_m = (k_{to} + \Delta k_t) i_m + M(x,t) \quad (5)$$

rearranging the terms

$$J_{mo} \ddot{\theta}_m = k_{to} i_m + M(x,t) - \Delta J_m \ddot{\theta}_m + \Delta k_t i_m - B \dot{\theta}_m \quad (6)$$

where  $J_{mo}$  and  $k_{to}$  are the nominal actuator inertia and torque constant, while  $\Delta J_m$  and  $\Delta k_t$  are the variation between nominal and actual actuator's inertia and torque constant. The last four terms of the right hand side of (6) can be considered as disturbance on the actuator

$$d = M(x,t) - \Delta J_m \ddot{\theta}_m + \Delta k_t i_m - B \dot{\theta}_m \quad (7)$$

that can be estimated through a first order low pass filter as follows

$$\hat{d} = \frac{g_{dist}}{s + g_{dist}} [g_{dist} J_{mo} \ddot{\theta}_m + k_{to} i_m] - g_{dist} J_{mo} \dot{\theta}_m \quad (8)$$

where  $g_{dist}$  is the low pass filter corner frequency or the positive observer gain. Replacing the actual with the estimated disturbance (7) can be written as follows

$$\widehat{M}(x,t) = \hat{d} + \Delta J_m \ddot{\theta}_m - \Delta k_t i_m + B \dot{\theta}_m \quad (9)$$

where  $\widehat{M}(x,t)$  is the bending moment estimate. In order to determine  $\widehat{M}(x,t)$  from (9) the last three terms have to be determined to decouple the bending moment out of the total disturbance  $\hat{d}$ . They are inherent properties of the actuator therefore they can be determined by an off line parameter identification experiment that is explained in the following sections. Using (2) along with the bending moment estimate of (9) we can show the following equation for a beam with a fixed end boundary condition

$$\widehat{y}(x,t) = \frac{1}{EI} [\widehat{M}(x,t) (\frac{x^2}{2} - lx + \frac{l^2}{2})] \quad (10)$$

where  $\widehat{y}(x,t)$  is the estimate of the beam's lateral displacement,  $l$  is beam's length. Equation (10) represent a position observer for any point  $x$  on the beam that can be obtained without taking any measurement from the flexible beam. Consequently, beam dynamics in both acceleration and velocity levels can be obtained by proper differentiation of (10) that is valid when the beam's tip interacts with the environment. In other words, (10) is valid for a fixed end beam's boundary condition that is perfectly suitable for an application such as micro manipulation of micro objects and biological cells, where a cantilever beam is used to orient these objects. Clearly, as soon as the cantilever interacts with the environment, the free end turns into fixed end boundary condition. Therefore, (10) can be used to estimate dynamics on configuration level, then acceleration can be obtained and used to estimate the interaction forces at the beam's tip.

### B. Lumped flexible Systems

1) *Disturbance estimation*: The actuator's mechanical equation of motion when its connected to a multi-degree-of-freedom flexible system with  $n$  degrees of freedom is

$$(J_{mo} + \Delta J_m) \ddot{\theta}_m = (k_{to} + \Delta k_t) i_m - \tau_{ref} \quad (11)$$

$$\tau_{ref} \triangleq B(\dot{\theta}_m - \dot{\theta}_1) - k(\theta_m - \theta_1) - f_c$$

re-arranging the terms and neglecting the coulomb friction  $f_c$

$$J_{mo} \ddot{\theta}_m = k_{to} i_m - \tau_{ref} - \Delta J_m \ddot{\theta}_m + \Delta k_t i_m \quad (12)$$

Where,  $\Delta J_m \ddot{\theta}_m$  and  $\Delta k_t i_m$  are the actuator's varied self-inertia torque and actuator's torque ripple. Therefore, the disturbance on the actuator side is

$$d = -\tau_{ref} - \underbrace{\Delta J_m \ddot{\theta}_m + \Delta k_t i_m}_{\text{disturbance}} \quad (13)$$

which indicates that disturbance on the actuator side is composed of two components. The last two terms of the right hand side of (13) represents the first disturbance component that is related to the actuator parameters's variations. While the second disturbance component  $\tau_{ref}$  is due to the attached system to this actuator. Therefore, disturbance  $d$  has to be

estimated then reflected torque load has to be decoupled out of it.

From (11) disturbance  $d$  can be computed as follows

$$d = J_{mo} \frac{d^2 \theta_m}{dt^2} - k_{to} i_m \quad (14)$$

or estimated through a low pass with a corner frequency  $g_{dist}$  [6]

$$\hat{d} = \frac{g_{dist}}{s + g_{dist}} [J_{mo} \ddot{\theta}_m - i_m k_{to}] \quad (15)$$

Therefore, the estimation error is

$$\tilde{d} = \hat{d} - d \quad (16)$$

introducing (14) and (15) into (16)

$$\tilde{d} = [J_{mo} \ddot{\theta}_m - i_m k_{to}] \frac{g_{dist}}{s + g_{dist}} - J_m \ddot{\theta}_m + i_m k_t \quad (17)$$

multiplying (15) by  $(s + g_{dist})$  and making the following definition

$$\xi \triangleq g_{dist} \Delta J \ddot{\theta}_m - s J_m \ddot{\theta}_m + g k_t i_m + s i_m k_t$$

we obtain the following differential equation

$$\frac{d}{dt} \tilde{d} + g_{dist} \tilde{d} = \xi \quad (18)$$

which describes the estimation error dynamics and has the following solution

$$\tilde{d}(t) = e^{-g_{dist} t} \int_0^t e^{g_{dist} \tau} \xi(\tau) d\tau + c e^{-g_{dist} t} \quad (19)$$

Therefore, the estimation error will exponentially decay and changing the observer gain controls the speed of the estimation convergence.

$$\begin{aligned} t \mapsto \infty &\implies \tilde{d} \mapsto 0 \\ \tilde{d} \mapsto 0 &\implies \hat{d} \mapsto d \end{aligned}$$

The direct differentiation of the velocity signal can be avoided by using the following observer configuration to keep the noise amplification level as low as possible [2]-[5].

$$\hat{d} = \frac{g_{dist}}{s + g_{dist}} [J_{mo} \dot{\theta}_m + k_{to} i_m] - g_{dist} J_{mo} \dot{\theta}_m \quad (20)$$

2) *Torque load decoupling:* Disturbance estimate obtained using (13), can be written as follows

$$\hat{\tau}_{ref} = -\hat{d} - \underbrace{\Delta J_m \ddot{\theta}_m + \Delta k_t i_m}_{\text{reflected torque}} \quad (21)$$

which indicates that in order to decouple the reflected load  $\hat{\tau}_{ref}$  out of the estimated disturbance, the self varied-inertia torque and actuator's torque ripple have to be determined first. Hence, variation between the actual and nominal actuator's parameters are inherent properties of the actuator. In other words, they are independent to the plant connected with the actuator. Therefore, they can be estimated from the unloaded actuator and in this case the reflected torque wave is eliminated from equation (11).

$$(J_{mo} + \Delta J_m) \ddot{\theta}_m + B \dot{\theta}_m = (k_{to} + \Delta k_t) i_m \quad (22)$$

$$J_{mo} \ddot{\theta}_m = k_{to} i_m + d_{par} \quad (23)$$

where  $d_{par}$  is the actuator's parameters variation disturbance

$$d_{par} = \Delta k_t i_m - \Delta J_m \ddot{\theta}_m - B \dot{\theta}_m \quad (24)$$

that can be estimated using the actuator's current and velocity through a low pass filter as follows

$$\hat{d}_{par} = \frac{g_{dist}}{s + g_{dist}} [J_{mo} \dot{\theta}_m + i_m k_{to}] - g_{dist} J_{mo} \dot{\theta}_m \quad (25)$$

equation (24) becomes

$$\hat{d}_{par} = -B \dot{\theta}_m + \Delta k_t i_m - \Delta J_m \ddot{\theta}_m \quad (26)$$

where,  $\hat{d}_{par}$  is the estimated parameters's disturbance vector data point, while  $\dot{\theta}_m$ ,  $\ddot{\theta}_m$  and  $i_m$  are data points vectors of actuator's velocity, acceleration and current. Putting (26) in the following matrix form

$$[\Delta k_t \quad -B \quad -\Delta J_m] \begin{bmatrix} i_m \\ \dot{\theta}_m \\ \ddot{\theta}_m \end{bmatrix}_{3 \times m} = [\hat{d}_{par}] \quad (27)$$

where  $m$  is the number of data points of each vector.

$$H \triangleq \begin{bmatrix} i_m \\ \dot{\theta}_m \\ \ddot{\theta}_m \end{bmatrix}_{3 \times m}$$

Data matrix  $H$  can be obtained from the actuator side by measuring it's current and estimating it's velocity and acceleration. Indeed, obtaining the acceleration signal will result in high amplification of the noise level. However, an appropriate differentiation techniques can be used in this level [10].

Equation (27) represents an over determined system, and it's solution has to minimize norm square of errors to obtain optimum values of  $\Delta k_t$  and  $\Delta J_m$ .

$$[\widehat{\Delta k_t} \quad -\widehat{B} \quad -\widehat{\Delta J_m}] = H^\dagger [\hat{d}_{par}] \quad (28)$$

where  $H^\dagger$  is the pseudo-inverse of  $H$ .  $\widehat{\Delta k_t}$  and  $\widehat{\Delta J_m}$  are the estimated actuator's torque ripple and varied self-inertia torque, respectively.

Rewriting (21) and replacing the actual parameter variations with the estimated ones, we get the estimated reflected torque load as follows

$$\hat{\tau}_{ref} = \widehat{\Delta k_t} i_m - \widehat{\Delta J_m} \ddot{\theta}_m - \hat{d} \quad (29)$$

The direct differentiation of the velocity signal can be avoided by using the following observer's structure

$$\hat{\tau}_{ref} = G(s) [i_m \widehat{\Delta k_t} - \hat{d} + g_{ref} \widehat{\Delta J_m} \dot{\theta}_m] - g_{ref} \widehat{\Delta J_m} \dot{\theta}_m \quad (30)$$

$$G(s) = \frac{g_{ref}}{s + g_{ref}}$$

where  $g_{ref}$  is reflected torque observer's constant gain.

3) *Parameters estimation:* Since system's stiffness and damping are inherent properties of the system. In other words, they are independent of the external applied torques. We assume that flexible system is free from external torques. Therefore, (11) can be written as

$$\hat{\tau}_{ref} = \sum_{i=1}^n J_i \ddot{\theta}_i = B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1) \quad (31)$$

This assumption is made to determine the system parameters through an off-line experiment. Then, the estimated parameters along with the estimated dynamics will be used in order to estimate the external torques/forces. However, determination of the system's uniform parameters  $k$  and  $B$  from (31) requires measuring the first mass's position. Surprisingly enough that if system's flexible

modes are not excited, a single generalized coordinate is enough to describe motion of the flexible system that is no longer flexible. Therefore, we can write

$$\theta_1(t) = \theta_2(t) = \theta_3(t) = \dots = \theta_n(t) = \theta(t) \quad (32)$$

this equality is valid if and only if the control input is filtered such that it contains zero energy at the system's resonance frequencies [12] or Fourier synthesized to guarantee that its frequency content at the system's resonances is zero. Moreover, shaping the input by this way makes it possible to estimate the rigid motion of the flexible system by the following equation

$$\hat{\theta}(t) = \frac{1}{\sum_{i=1}^n J_i} \int_0^t \int_0^t \hat{\tau}_{ref} d\tau d\tau + c_1 t + c_2 \quad (33)$$

where  $\hat{\theta}(t)$  is the rigid motion position estimate. equation (33) is only valid through a narrow region of the flexible system's frequency range. Therefore, within this frequency range the parameters estimation process has to be performed.

Rewriting (31) and using  $\hat{\theta}(t)$  instead of  $\theta_1(t)$

$$\hat{\tau}_{ref} = B(\dot{\theta}_m - \dot{\hat{\theta}}) + k(\theta_m - \hat{\theta}) \quad (34)$$

then defining

$$\underline{\xi} \triangleq (\theta_m - \hat{\theta}) \quad , \quad \underline{\eta} \triangleq (\dot{\theta}_m - \dot{\hat{\theta}})$$

where  $\underline{\xi}$  and  $\underline{\eta}$  are vectors of data points. Similarly,  $\hat{\tau}_{ref}$  is the estimated reflected torque data point vector, rewriting (34) in the following matrix form

$$\begin{bmatrix} \underline{\xi} & \underline{\eta} \end{bmatrix}_{n \times 2} \begin{bmatrix} k \\ B \end{bmatrix}_{2 \times 1} = \begin{bmatrix} \hat{\tau}_{ref} \end{bmatrix}_{n \times 1} \quad (35)$$

$$G \triangleq \begin{bmatrix} \underline{\xi} & \underline{\eta} \end{bmatrix}$$

solving (35) for the system parameters vector we obtain

$$\begin{bmatrix} \hat{k} \\ \hat{B} \end{bmatrix} = G^\dagger \begin{bmatrix} \hat{\tau}_{ref} \end{bmatrix} \quad (36)$$

Where  $G^\dagger$  is the pseudo inverse of  $G$ .  $\hat{k}$  and  $\hat{B}$  are the estimates of the system's uniform stiffness and damping coefficients, respectively. The previous procedure is considered as an off-line parameters estimation experiment that is performed in a certain system's frequency range to estimate flexible system's uniform stiffness and damping.

### III. EXTERNAL TORQUE ESTIMATION

In the presence of externally applied torque due to system interaction with the environment, (31) can be rewritten as follows

$$\hat{\tau}_{ref} = \sum_{i=1}^n J_i \ddot{\theta}_i - \sum_{i=1}^n \tau_{ext_i} \quad (37)$$

where,  $\tau_{ext_i}$  is the externally applied torque on the  $i^{th}$  lumped mass. Therefore, in order to extract the interaction torque information from (37), acceleration level dynamics of the flexible plant have to be available.

#### A. Sensorless States Estimation

Equation (33) is valid in narrow region of the system's frequency range. In addition, it estimates the motion of the flexible system when its rigidly behaving. Therefore, (33) is not enough to estimate the flexible system's motion, where the amplitude ratios between the masses are no longer unity and masses are no longer in phase.

Recalling (31) and replacing the actual parameters with the estimated ones, we obtain the following differential equation

$$\hat{B}\dot{\theta}_1 + \hat{k}\theta_1 = \hat{B}\dot{\theta}_o + \hat{k}\theta_o - \hat{\tau}_{ref} \quad (38)$$

solving the first order differential equation for  $\theta_1(t)$  that can be denoted as  $\hat{\theta}_1(t)$  since it depends on observed variable  $\hat{\tau}_{ref}$  and estimated parameters  $\hat{k}$  and  $\hat{B}$ .

$$\hat{\theta}_1(t) = e^{-\frac{\hat{k}}{\hat{B}}t} \int_0^t \beta e^{\frac{\hat{k}}{\hat{B}}\tau} d\tau + e^{-\frac{\hat{k}}{\hat{B}}t} c_1 \quad (39)$$

$$\beta \triangleq \frac{\alpha}{\hat{B}} \quad , \quad \alpha \triangleq \hat{B} \dot{\theta}_o + \hat{k} \theta_o - \hat{\tau}_{ref}$$

$\hat{\theta}_1(t)$  is the position estimate of the first mass and (39) is valid through the entire system's frequency range regardless to the frequency content of the forcing function.

Recalling the first flexible system's equation of motion and replacing the first mass position with its estimate we obtain

$$\hat{B} \dot{\theta}_2 + \hat{k} \theta_2 = J_1 \hat{\theta}_1 - \hat{B}(\dot{\theta}_o - \dot{\hat{\theta}}_1) - \hat{k}(\theta_o - \theta_1) + \hat{B} \hat{\theta}_1 + \hat{k} \hat{\theta}_1 \quad (40)$$

solving for  $\hat{\theta}_2(t)$  we obtain

$$\hat{\theta}_2(t) = e^{-\frac{\hat{k}}{\hat{B}}t} \int_0^t \zeta e^{\frac{\hat{k}}{\hat{B}}\tau} d\tau + e^{-\frac{\hat{k}}{\hat{B}}t} c_2 \quad (41)$$

$$\zeta \triangleq \frac{\gamma}{\hat{B}} \quad , \quad \gamma \triangleq J_1 \hat{\theta}_1 - \hat{B}(\dot{\theta}_o - \dot{\hat{\theta}}_1) - \hat{k}(\theta_o - \theta_1) + \hat{B} \hat{\theta}_1 + \hat{k} \hat{\theta}_1$$

In general, the position estimate of the  $i^{th}$  lumped mass is

$$\hat{\theta}_i(t) = e^{-\frac{\hat{k}}{\hat{B}}t} \int_0^t \Omega e^{\frac{\hat{k}}{\hat{B}}\tau} d\tau + e^{-\frac{\hat{k}}{\hat{B}}t} c_i \quad (42)$$

$$\Omega \triangleq \frac{\Psi}{\hat{B}} \quad , \quad \Psi \triangleq g(J_{i-1}, \hat{\theta}_{i-1}, \hat{\theta}_{i-1}, \hat{\theta}_{i-1}, \hat{k}, \hat{B})$$

Equations<sup>1</sup> (42) can be considered as a set of position observers that are recursively estimating positions of system's lumped masses. Surprisingly enough that if a proper differentiation technique [10] is used, dynamics in both velocity and acceleration levels can be obtained without taking any measurement from the flexible plant.

#### B. Compensation Function

Its obvious that each state observer (42) includes an integrator to compute estimate of each lumped mass position. The integration process however will magnify any initial error with time, results in divergence of the estimates from the actual values. Therefore, a compensation function is added to prevent accumulation of error during the integration process.

Assuming that we have a plant with the following structure

$$J_m \ddot{\theta}(t) = i_m(t) k_t \quad (43)$$

where  $J_m$  and  $k_t$  are plant parameters, while  $i_m(t)$  is a time varying signal. Solving for  $\theta_m(t)$  we obtain

$$\theta_m(t) = \int_{t_o}^{t_f} \int_{t_o}^{t_f} \frac{k_t}{J_m} i_m(\tau) d\tau d\tau \quad (44)$$

<sup>1</sup>These recursive equations are estimating position of each lumped mass but they do not guarantee the convergence of the estimates to the actual values, however the compensation function presented in the section B can be used to prevent divergence of the estimated states.

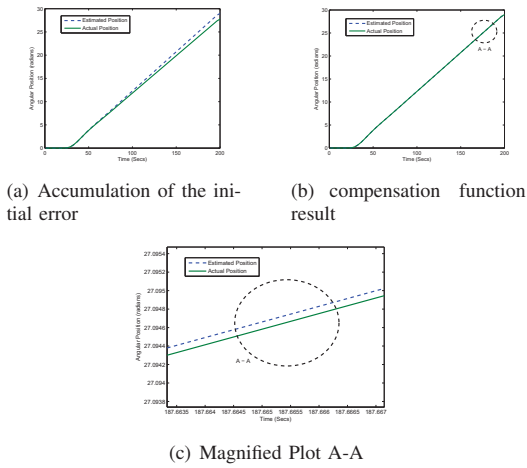


Fig. 1. Error Compensation Results

which cannot be used to compute the signal  $\theta_m(t)$  as the single or double integration action will accumulate any initial error with time. Fig.1.a illustrates the divergence of  $\theta_m(t)$  from the actual one due to the double integration action of (44), making the following definition

$$\Gamma \triangleq \frac{k_t}{J_m}$$

where  $\Gamma$  represents the parameter ratio

$$\Gamma = \Gamma_o + \Delta\Gamma \quad (45)$$

$\Gamma_o$  is the actual parameters ratio, while  $\Delta\Gamma$  is the deviation from the this ratio. The double integration action acts on the product of the constant  $\Delta\Gamma$  with the time varying signal  $i_m(\tau)$  results on the divergence illustrated in Fig.1.a. No matter how small the initial error  $\Delta\Gamma$  is, multiplication with a time varying signal along with the integration will definitely results in a divergence of the position estimate.

Indeed, estimating some states and proving their convergence to the actual values requires taking some measurements from the system to be used as basis of the estimation process, however this work attempts to keep the plant free from any measurement. Therefore, the single integration action of (42) is investigated to demonstrate whether it can be used as an observer. Using the linearity principle of integration, (44) can be rewritten as follows

$$\theta_m(t) = \int_{t_o}^{t_f} \int_{t_o}^{t_f} \Gamma_o i_m(\tau) d\tau d\tau + \int_{t_o}^{t_f} \int_{t_o}^{t_f} \Delta\Gamma i_m(\tau) d\tau d\tau \quad (46)$$

where the second term of (46) is the accumulated error. Surprisingly enough that this term is mainly depending on the time varying signal. In other words, the time varying signal has the largest contribution on the accumulated error unlike the constant  $\Delta\Gamma$  that indicates the difference between the actual and nominal parameters. Introducing the following compensation function

$$F(t) = \Lambda \int_{t_o}^{t_f} \int_{t_o}^{t_f} i_m(\tau) d\tau d\tau \quad (47)$$

where,  $\Lambda$  is an uncertainty factor that depends on the closeness of the actual and nominal parameters. In other words, if the system parameters are already known beforehand, then  $\Lambda = 0$ . The actuator position estimate can be obtained using the following formula

$$\hat{\theta}_m(t) = \int_{t_o}^{t_f} \int_{t_o}^{t_f} \Gamma i_m(\tau) d\tau d\tau - F(t) \quad (48)$$

Fig.1.b-c illustrates the result of (48). It turns out that the compensation function eliminates the divergence of the estimate from

the actual value<sup>2</sup>, however there exists a steady state error in the final response. Fig.2 illustrates the block diagram implementation of (48).

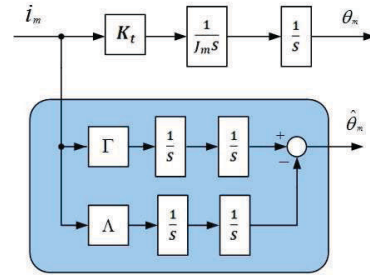


Fig. 2. Double integration with compensating function

### C. External Torque Estimation

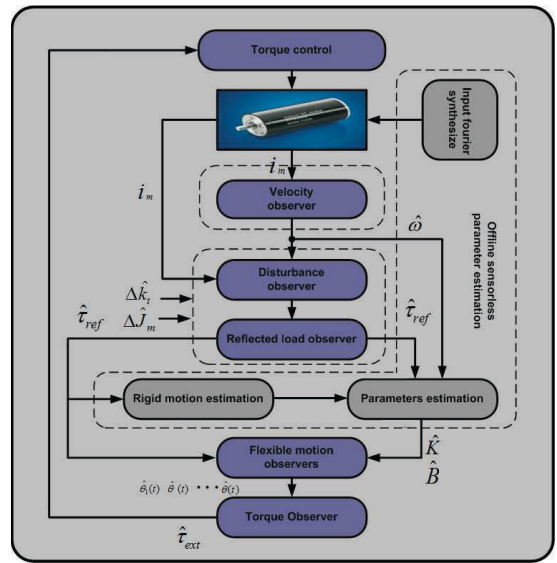


Fig. 3. Overall Torque estimation scheme

Since the flexible system's dynamics can be estimated using (42) along with the reflected torque load (21), equation (37) can be used in order to estimate the externally applied torques due to system's interaction with the environment as follows.

$$\hat{\tau}_{ext} = \sum_{i=1}^n J_i \hat{\theta}_i - \hat{\tau}_{ref} \quad (49)$$

Where  $\hat{\tau}_{ext}$  is the estimate of the external applied torques on the plant.

The sensorless torque estimation process is illustrated in Fig.3. Indeed, the word 'Sensorless' is not precise, since one must sense or measure some variable to obtain some information as a basis of estimating the unknown variables. In this context the word 'sensorless' refers to the flexible plant that is kept free from any measurement and only two measurement are required from the actuators side as shown in Fig.3. Where actuator's current and

<sup>2</sup>It is important to emphasize that convergence of the estimated values to the actual values are not possible due to the open loop kind of structure, however the compensating function prevents the divergence of the estimates.

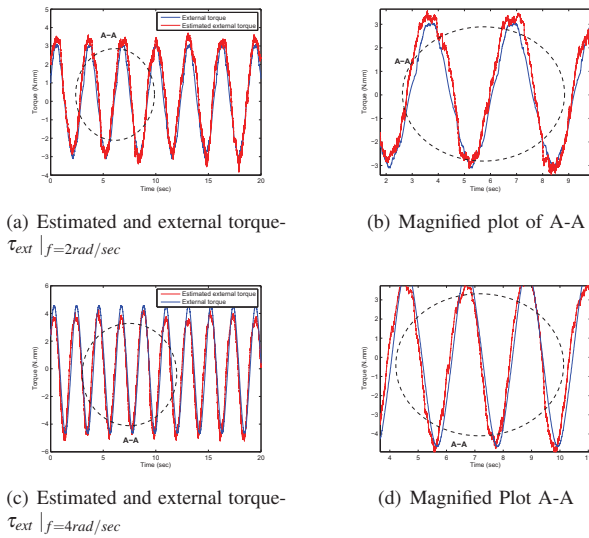


Fig. 4. External torque estimation

TABLE I  
EXPERIMENTAL PARAMETERS

Parameter	Value	Parameter	Value
$J_1$	5152.9 gcm <sup>2</sup>	$J_3$	6192.7 gcm <sup>2</sup>
$J_2$	5152.9 gcm <sup>2</sup>	$f_1$	1 rad/sec
$f_2$	2 rad/sec	$f_3$	3 rad/sec
$f_4$	4 rad/sec	$k_{to}$	40.6 mNm/A
$k_b$	235 rpm/v	$J_{mo}$	209 gcm <sup>2</sup>
$g_{dist}$	100 rad/sec	$g_{tpf}$	100 rad/sec

velocity are measured and used to estimate the total disturbance, then the reflected torque wave is decoupled out of this disturbance and used as an input for a chain of flexible motion observers that provide (42) with the necessary entries to estimate the external torque. The previous procedure requires performing a couple of off-line experiments based on actuator's current and velocity. The first experiment is performed to estimate the parameters variation disturbance in order to decouple the reflected torque load out of the total disturbance, while the second experiment is performed in order to estimate the system's uniform parameters.

#### D. Experimental Results

Estimating the external torque applied on the flexible system using (49) requires estimating the flexible system's dynamics and estimate of the reflected torque wave. Experimentally a sinusoidal torque disturbance is added to the flexible system and simply measured and compared with the estimated one. Experimental parameters are included in Table.I. The frequency of the externally applied sinusoidal torque was varied between 1-5 rad/sec. Fig.4-a shows the estimated and actual torque of 2 rad/sec frequency, while Fig.4-c illustrates the same result when the external torque's frequency is 4 rad/sec.

#### IV. CONCLUSION

This paper introduces an algorithm for estimating the externally applied disturbance due to flexible system's interaction with the environment and consequently controlling torques at system's task space without using any force sensor.

The proposed algorithm is treating the disturbances and system's flexibility differently. Disturbance is not only used as a step toward accomplishment of robust motion control system but treated as

coupled signal that contains information about system's parameters, dynamics and external forces/torques. In addition, flexibility is treated as an efficient tool by shaping the control input to obtain certain behavior of the multi-degree-of-freedom system. In other words, making it possible to minimize number of generalized-coordinates describing flexible system's motion, that in turn makes it possible to determine system parameters easier as too many unknowns are dropped.

The reflected mechanical waves are decoupled out of the total disturbance and proved to contain enough information about the dynamical system. Then, system is rigidly excited with a pre-filtered control input to estimate system's rigid motion. Moreover, rigid system's position is used along with reflected torque wave estimate to estimate the uniform system's stiffness and damping with less than 5 percent error when compared with the actual parameters that are known before hand. Then, the estimated parameters are used along with the reflected mechanical waves and actuator parameters to design a chain of flexible motion observers that are recursively estimating flexible motion of each lumped mass of the system. Proper differentiation of the position estimates makes all system's dynamics available. Eventually, the estimated dynamics along with the reflected torque waves are used to estimate the externally applied torque. Experimental results show the validity of the algorithm that can be used in order to accomplish a sensorless force control assignments.

#### V. ACKNOWLEDGMENTS

The authors gratefully acknowledge Yousef Jameel scholarship program and The Scientific and Technological Research Council of Turkey (TUBITAK) for the financial support.

#### REFERENCES

- [1] Y. Ohba, S. Katsura and K. Ohinhi *Sensor-less Force Control for Machine Tool Using Reaction Torque Observer*, IEEE, Vol-N1, pp.860-866, 2006.
- [2] T. Murakami, F. Yu and K. Ohnisi, *Torque Sensorless Control in Multidegree-of-Freedom Manipulator*, IEEE Transaction On Industrial Electronics, Vol 40 - N2, pp.259-256, 1993.
- [3] W. Iida and K. Ohinhi *Sensorless Force Control with Force Error Observer*, IEEE, ICIT *International Conference on Industrial Electronics*, Maribor, Slovenia, pp.157-162, 2003.
- [4] F. Yu, T. Murakami and K. Ohinhi *Sensorless Force Control of Direct Drive Manipulator*, In Proc IEEE, pp.311-315.
- [5] M. Sugita, T. Murakami and K. Ohnisi, *Following Control of Flexible Manipulator*, IEEE AMC, Vol 1-N1, pp.47-51, 2000.
- [6] K. Ohnisi and M. Shibata, *Motion Control for Advanced Mechatronics*, IEEE, Vol 1 - N1, pp.56-67, 1996.
- [7] H. haung, D. Sun, J. K. Mills and W. J. Li *Visual-based Impedance Control of Three-dmensional cell injection system*, In Proc IEEE, ICRA *International Conference on Robotics and Automation*, Roma, Italy, pp.4196-4210, 2007.
- [8] Islam S. M. Khalil, E. D. Kunt and Asif Sabanovic, *Estimation Based PID Controller-Sensorless Wave Based Technique*, IFAC International Conference on Intelligent Control System and Signal Processing, Vol 1-N1, pp.101-106, 2009.
- [9] Islam S. M. Khalil, E. D. Kunt and Asif Sabanovic, *Sensorless Torque Estimation For Multi-degree-of Freedom Systems*, IECON The 35th Annual Conference of IEEE Industrial Electronic Society, 2009.
- [10] T. Tsuji, M. Mizuochi, H.ishi and K. Ohnisi, *A Velocity Measurement Method for Acceleration Control*, In Proc IEEE, Vol 1-N1, pp.1943-1947, 2005.
- [11] D. K .Miu, *Mechatronics - Electromechanics and Contromechanics*, 3rd ed. New York, USA: Springer-Verlag, 1993.
- [12] S. P. Bhat and D. K. Miu, *Solutions to Point-to-Point Control Problems using Laplace Transform Technique*, ASME, Vol 113-N13, pp.425-431, 1991.
- [13] S. Khan, A. O . Nergiz, M. Elitas, V. Patoglu and A. Sabanovic, *A Hybrid Force-Position Controller Based Man-Machine Interface for manipulation of Micro Objects*, In Proc IEEE, pp.62-67, 2007.
- [14] C. T. Chen, *Linear System Theory and Design*, 3rd ed. New York, USA: Oxford, 1999.