Action-Reaction Based Parameters Identification and States Estimation of Flexible systems

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***Abstract*—This work attempts to identify and estimate flexible system's parameters and states by a simple utilization of the Action-Reaction law of dynamical systems. Attached actuator to a dynamical system or environmental interaction imposes an action that is instantaneously followed by a dynamical system reaction. The dynamical system's reaction carries full information about the dynamical system including system parameters, dynamics and externally applied forces that arise due to system interaction with the environment. This in turn implies that the dynamical system's reaction can be considered as a natural feedback as it carries full coupled information about the dynamical system. The idea is experimentally implemented on a dynamical system with three flexible modes, then it can be extended to the more complicated structures with infinite flexible modes.**

I. Introduction

It is commonly believed that robust motion control can be achieved by estimating the incident disturbances that arise due to an action imposed either intentionally by the actuator or unintentionally by system's interaction with the environment then converting the estimated disturbance into additional control input that eliminates these disturbances in an inner loop of the control system [1]-[4]. Load torque, externally applied torques or forces due to system interaction with the environment and model uncertainties are the main components of the disturbance signal where the load torque depends mainly on the dynamical system attached to the actuator and its mathematical expression can be obtained through system's model [10]. The reflected torque definitely however is nothing but the instantaneous reaction of the dynamical system to any action imposed by the actuator. In other words, at the interface point where both the actuator and the dynamical system coincide, the action and the instantaneous reaction events occur. Consequently, a mathematical expression of the reaction signal can be developed based on the knowledge of the system's dynamical model. Moreover, the reaction signal can be estimated along with other signals through a disturbance observer that utilizes actuator measurements, namely actuator's current and velocity [2]. Furthermore, the reaction signal includes coupled information about system parameters such as damping coefficients and joints stiffness along with acceleration level system's dynamics and environmental interaction torques or forces. In other words, dynamical system's instantaneous reaction can be considered as a natural feedback. The natural feedback concept was presented by O'Connor [5]-[6], where the actuator was used to launch mechanical waves to the system and to absorb the incident waves to keep the system free from residual vibration after a motion assignment maneuver [7]. In this work, the incident torque load is considered as an instantaneous reaction of the dynamical system which can be estimated using the actuator's current and velocity then analyzed to extract system parameters and states. Estimated parameters and states can then be used to perform a motion and vibration control assignment without taking any measurement from the flexible plant.

This paper is organized as follows, Section II includes a derivation of a mathematical expression for the reaction signal and the incident disturbances for flexible system with finite number of degrees of freedom then the work can be extended to the more complicated systems with infinite modes. Section III includes a parameters identification and states estimation algorithm that differs from the existing techniques in the sense of keeping the dynamical system free from any measurement. However, only two measurements are required to be taken from the actuator side. Experimental results of the proposed algorithm are included in section IV. Eventually, conclusions and final remarks are included in section V.

II. Action-Reaction Approach

The state space model for a linear time invariant system can be written as follows

$$\dot{\mathrm{x}} = \mathrm{Ax} + \mathrm{b}u + \mathrm{ed}' \quad , \quad y = \mathrm{cx}. \tag{1}$$

Where x and y are states and outputs vectors. A, b, c and e are system matrix, distribution vector of the input, observation column vector and distribution vector of the disturbance d' respectively.

Considering the parameters variation

$$\mathrm{A} = \mathrm{A}_o + \triangle\mathrm{A} \quad , \quad \mathrm{b} = \mathrm{b}_o + \triangle\mathrm{b} \tag{2}$$

$\triangle\mathrm{A}$ and $\triangle\mathrm{b}$ are the deviations from (A, b) and their nominal values (A_o, b_o), respectively. The new state space equations therefore are

$$\begin{aligned} \dot{\mathrm{x}} &= (\mathrm{A}_o + \triangle\mathrm{A})\mathrm{x} + (\mathrm{b}_o + \triangle\mathrm{b})u + \mathrm{ed}' \\ &= \mathrm{A}_o\mathrm{x} + \mathrm{b}_o u + (\triangle\mathrm{A}\mathrm{x} + \triangle\mathrm{b}u + \mathrm{ed}') \end{aligned} \tag{3}$$

The third term of the right hand side of (3) represents both the instantaneous reaction signal and parameter variation disturbance

$$d \triangleq \triangle \mathrm{A}x + \triangle \mathrm{b}u + \mathrm{e}d' \tag{4}$$

Applying the previous equations on the dynamical system illustrated in Fig.1 which consists of an inertial multi-degree of freedom system with uniform damping coefficient B and stiffness k. i_a, k_t, θ_m and θ_i are the actuator's current, torque constant, angular position and dynamical system's coordinates, respectively.

$$\begin{aligned} d &= \tau_{reac} + \triangle k_t i_m - \triangle J_m \ddot{\theta}_m \quad (5)\\ &= k(\theta_m - \theta_a) + B(\dot{\theta}_m - \dot{\theta}_a) + \triangle k_t i_m - \triangle J_m \ddot{\theta}_m \end{aligned}$$

τ_{reac}(t) is the instantaneous reaction torque load that can be expressed for the dynamical system illustrated in Fig.1 as follows

$$\begin{aligned} \tau_{reac}(t) &\triangleq B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1) \quad (6)\\ &\triangleq \sum_{i=1}^{n} J_i \ddot{\theta}_i - \sum_{i=1}^{n} \tau_{ext_i} \end{aligned}$$

Indeed, the model illustrated in Fig.1 is simple and doesn't represent the more practical systems with infinite modes such as flexible manipulators and beams. However, the following equation represents the reaction torque from a flexible beam on the interface point with the actuator [8]

$$\tau_{reac}(t,0) = EI\frac{\partial^{2y(t,x)}}{\partial x^2} = \int_0^L \int_0^L \Phi(t,0)\mathrm{dxdx} + c_1\mathrm{x} + c_2$$

$$\Phi(t,\mathrm{x}) \triangleq \tau(t,\mathrm{x}) - B\frac{\partial y(t,\mathrm{x})}{\partial t} - \rho A \frac{\partial^{2y(t,\mathrm{x})}}{\partial t^2} \tag{7}$$

Where E, I, ρ, L and A are the flexible manipulator's modulus of elasticity, moment of inertia, density, length and cross sectional area, while $y(t,x)$ and $\tau(t,0)$ are the manipulator's lateral displacement and actuator's input torque, c_1 and c_2 are integration constants, respectively. Equation (6) represents the reaction torque of the lumped flexible system illustrated in Fig.1, which in turn implies that system parameters along with system dynamics in the acceleration level and externally applied torques τ_{ext} are coupled in the incident reaction torque τ_{reac}. Similarly, (7) represents the reaction torque of a flexible manipulator to an action imposed by an actuator located at $x = 0$. Nevertheless, this paper is concerned with lumped dynamical system. Therefore, (6) is used in the attempt to estimate system parameters and dynamics through two measurement taken from the actuator[1]. Consequently, disturbance $d(t)$ can be estimated from the actuator side by writing the actuator mechanical equation of motion as follows

$$J_{mo}\frac{d^2\theta_m}{dt^2} = k_{to} i_a - d(t) \tag{8}$$

[1]Actuator current and velocity are measured while rest of the dynamical system is kept free from any measurement considering the reaction signal as a natural feedback from the system.

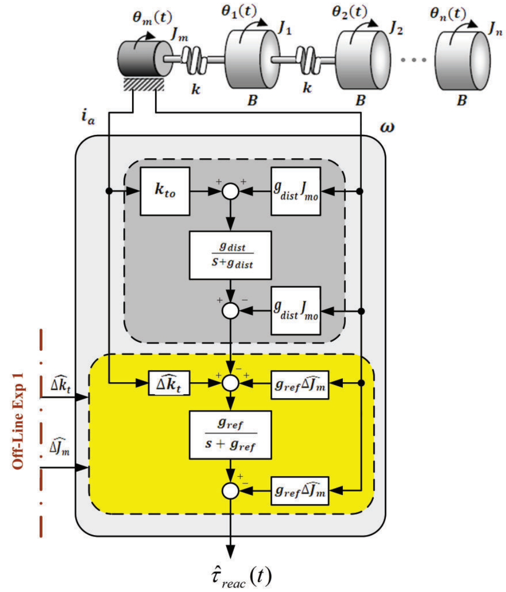


Fig. 1. Disturbance and reaction torque observers.

$$d(t) = B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1) + f_{cm} - \triangle k_t i_a + \triangle J_m \frac{d^2\theta_m}{dt^2}$$

Where, J_{mo}, k_{to} and f_{cm} are the nominal actuator inertia, torque constants and coulomb friction. ΔJ_m and Δk_t are the variations between actuator's nominal and actual values, respectively. Disturbance d can be estimated through the following low pass filter with a corner frequency $g_{dist} \in \mathrm{R}^+$ [4]

$$\widehat{d}(t) = \mathrm{G}(s)[g_{dist}J_{mo}\frac{d\theta(t)}{dt} + i_a(t)k_{to}] - g_{dist}J_{mo}\frac{d\theta(t)}{dt} \tag{9}$$

$$\mathrm{G}(s) = \frac{g_{dist}}{s + g_{dist}}$$

Therefore, the estimation error can be computed as follows

$$\widetilde{d} = \widehat{d}(t) - d(t) \tag{10}$$

$$\widetilde{d} = \mathrm{G}(s)[g_{dist}J_{mo}\dot{\theta}(t) + i_a(t)k_{to}] - g_{dist}J_{mo}\dot{\theta}(t) - J_{mo}\ddot{\theta}(t) + k_{to}i_a(t)$$

Consequently, the disturbance error dynamics is governed by the following differential equation

$$\frac{d}{dt}\widetilde{d}(t) + g_{dist}\widetilde{d}(t) = \Omega(t) \tag{11}$$

$$\Omega(t) \triangleq g_{dist}^2 J_{mo}\dot{\theta}(t) + g_{dist} i_a(t) k_{to} + (s + g_{dist})\chi$$

$$\chi \triangleq k_t i_a - J_m \ddot{\theta}(t) - g_{dist} J_{mo}\dot{\theta}(t)$$

solving (11) for $\widetilde{d}(t)$ we obtain

$$\widetilde{d}(t) = c3\ e^{-g_{dist}t} + e^{-g_{dist}t}\int_o^T e^{-g_{dist}t}\Omega(t)\ dt \tag{12}$$

which guarantees the exponential convergence of the estimated disturbance to the actual one by the proper selection of the observer gain g_{dist}. In other words, as $t \to \infty \Rightarrow \widetilde{d}(t) \to 0 \Rightarrow \widehat{d}(t) \to d(t)$. The first block of Fig.1 illustrates the implementation of (9), where the actuator current and velocity are measured and used as inputs to the disturbance observer. However, in order to compute the reaction torque $\tau_{reac}(t)$ through (5), the varied self-inertia torque $\triangle J_m \ddot{\theta}_m(t)$ and the actuator torque ripple $\triangle k_t i_m(t)$ have to be determined, then subtracted out of $\widehat{d}$(t) so as to estimate the reaction torque $\tau_{reac}(t)$. Surprisingly enough that both actuator torque ripple and varied self-inertia torque are inherent properties of the actuator. In other words, they can be computed from the actuator when it is running free from any attached load. That in turn eliminates the reaction torque term $\tau_{reac}(t)$ from (5), consequently it can be written as follows

$$\underline{\widehat{d}}_{par}(t) = \underbrace{\tau_{reac}(t)}_{0} + \triangle k_t \underline{i}_m - \triangle J_m \underline{\ddot{\theta}}_m(t) - D\underline{\dot{\theta}}_m(t) \quad (13)$$

Where, $d(t)$ becomes $\widehat{d}_{par}(t)$ as the disturbance became dependent only on the parameters uncertainties as the actuator became free from any attached load whatsoever, $\tau_{reac}(t) = 0$. D is the viscous damping coefficient of the actuator. Putting (13) into the following over-determined matrix form

$$\begin{bmatrix} \triangle k_t & -D & -\triangle J_m \end{bmatrix}_{1\times 3} \begin{bmatrix} \underline{i}_m \\ \underline{\dot{\theta}}_m \\ \underline{\ddot{\theta}}_m \end{bmatrix}_{3\times r} = \begin{bmatrix} \widehat{d}_{par} \end{bmatrix}_{r\times 1}$$

$$\mathrm{H} \triangleq \begin{bmatrix} \underline{i}_m & \underline{\dot{\theta}}_m & \underline{\ddot{\theta}}_m \end{bmatrix} \quad (14)$$

Where $\underline{i}_m(t)$, $\underline{\dot{\theta}}_m(t)$ and $\underline{\ddot{\theta}}_m(t)$ are vectors of actuator's current, velocity and acceleration with r data points. Consequently, the optimum $\triangle k_t$ and $\triangle J_m$ can be determined as follows through (15)

$$\begin{aligned} \begin{bmatrix} \widehat{\triangle k_t} & -\widehat{D} & -\widehat{\triangle J_m} \end{bmatrix} &= \begin{bmatrix} \mathrm{H}^T\mathrm{H} \end{bmatrix}^{-1} \mathrm{H}^T \begin{bmatrix} \widehat{d}_{par} \end{bmatrix} \\ &= \mathrm{H}^{\dagger} \begin{bmatrix} \widehat{d}_{par} \end{bmatrix} \end{aligned} \quad (15)$$

Where, $\mathrm{H}^{\dagger}$ is the pseudo inverse of H. Using (14) along with (5), estimate of the incident reaction torque can be determined as follows

$$\widehat{\tau_{reac}}(t) = \widehat{d}(t) - \widehat{\triangle k_t} i_m(t) + \widehat{\triangle J_m} \ddot{\theta}_m(t) \quad (16)$$

Where, $\widehat{\tau_{reac}}(t)$ is the estimate of the instantaneous reaction of the dynamical system that arise due to an action imposed by either the actuator or by any kind of environmental interaction. Figure.1 illustrates the block diagram implementation of the reaction torque observer (16), where two actuator measurement are taken to estimate the disturbance $\widehat{d}(t)$, then an off-line experiment is performed to estimate both $\triangle k_t$ and $\triangle J_m$ in order to decouple $\widehat{\tau_{reac}}(t)$ out of $\widehat{d}(t)$.

III. Parameters Identification and States Estimation

A. Parameters Identification

Since the reaction torque is estimated using two actuator's measurements, (6) can be rewritten as follows

$$\widehat{\tau_{reac}}(t) \triangleq B(\dot{\theta}_m - \dot{\theta}_1) + k(\theta_m - \theta_1). \quad (17)$$

which indicates that, in order to estimate the uniform viscous damping coefficient and the uniform joints stiffness, angular position of the first inertial mass has to be measured. We already assumed that actuator angular velocity is available along with the estimate of the reaction torque. Therefore, one measurement from the dynamical system is required to be taken in order to determine B and k through (17). However, taking this measurement from the system will violate the natural feedback concept. The natural feedback concept naturally assumes that the dynamical system makes an instantaneous reaction that includes all system information that can be verified through (6) for lumped systems or (7) for continuous flexible systems due to any action imposed by the actuator or the external environment. Furthermore, we attempt to use this natural feedback or the incident reaction torque as an alternative to any attached sensor to the system in order to keep the dynamical system free from any measurement.

Surprisingly enough that system flexibility which is commonly believed to be a challenging control subject, can be used to keep the flexible system free from any measurement. Flexible systems have different behavior along their entire frequency range. In other words, for any given flexible system, a rigid relation between the lumped masses can be obtained in the low frequency range which is not the case for the rest of the frequency range as lumped masses moves with respect to each other with different amplitude and phase. Modal decomposition shows the relative relations between system's lumped masses at particular frequencies, namely the system's natural frequencies. For a system with (n) degrees of freedom, there exists a single rigid mode along with $(n-1)$ flexible modes [12]-[13]. A single generalized coordinate is required to describe motion of the system if none of its $(n-1)$ [8] flexible modes is excited. Definitely, such motion can be obtained if the control input does not contain any energy at the system resonances that can be accomplished by fourier synthesis of the control input so as to avoid exciting system's flexible modes. Another way to obtain the same rigid behavior is to filter the control input so as to ensure that it does not contain energy at the system resonances. The governing equations for a single input structure with one rigid mode and (n) flexible modes is of the following form

$$\begin{bmatrix} \dot{\theta}_0 \\ \ddot{\theta}_0 \\ \dot{\theta}_1 \\ \ddot{\theta}_0 \\ \vdots \\ \ddot{\theta}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -\omega_1^2 & -2\zeta_1\omega_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & \vdots \\ 0 & 0 & 0 & \vdots & -\omega_n^2 & -2\zeta_n\omega_n \end{bmatrix} \begin{bmatrix} \theta_0 \\ \dot{\theta}_0 \\ \theta_1 \\ \dot{\theta}_0 \\ \vdots \\ \dot{\theta}_n \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ \vdots \\ 1 \end{bmatrix} u$$

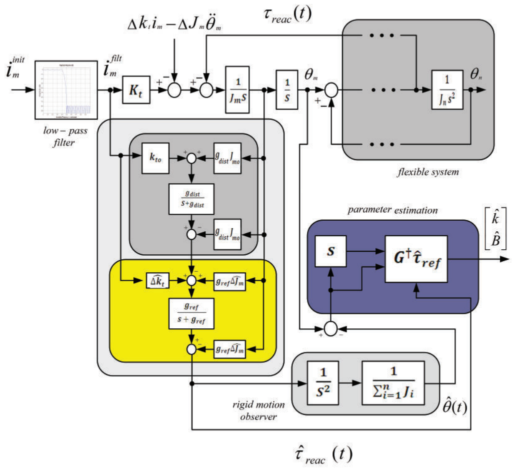


Fig. 2. Reaction torque observer and parameters estimation.

$$y = \begin{bmatrix}\phi_0 & 0 & \phi_1 & 0 & \cdots & 0\end{bmatrix}\begin{bmatrix}\theta_0\\ \dot{\theta}_0\\ \theta_1\\ \dot{\theta}_0\\ \vdots\\ \dot{\theta}_n\end{bmatrix} \tag{18}$$

Where, $\theta_0(t)$ is the rigid mode, while $\theta_0(t)\dots\theta_n(t)$ are the flexible modes. $\omega_1\dots\omega_n$ are the corresponding natural frequencies, $\zeta_1\dots\zeta_n$ and $\phi_1\dots\phi_n$ are the corresponding damping ratios and mode shapes, respectively [9]. Therefore, if the control input was filtered so as not to excite any of the system's flexible modes, the following equality can be obtained

$$\theta_1(t) = \theta_2(t) = \theta_3(t) = \dots = \theta_n(t) \tag{19}$$

consequently, the rigid motion of the flexible system can be described as follows

$$\widehat{\Theta}(t) = \frac{1}{\sum_{i=1}^{n} J_i}\int_0^t\int_0^t \widehat{\tau_{reac}}(t)d\tau d\tau + c_4 t + c_5 \tag{20}$$

Using $\widehat{\Theta}(t)$ instead of $\theta_1(t)$ and defining $\underline{\xi} \triangleq (\theta_m - \widehat{\Theta})$, $\underline{\eta} \triangleq (\dot{\theta}_m - \dot{\widehat{\Theta}})$, $\mathrm{G} \triangleq [\ \underline{\xi} \quad \underline{\eta}\ ]$. Therefore, the estimated system uniform damping coefficient and stiffness can be computed as follows

$$\begin{bmatrix}\widehat{k}\\ \widehat{B}\end{bmatrix} = [\ \mathrm{G}^T\mathrm{G}\ ]^{-1}\mathrm{G}^T [\ \widehat{\underline{\tau}_{reac}}\ ] = \mathrm{G}^{\dagger} [\ \widehat{\underline{\tau}_{reac}}\ ] \tag{21}$$

Where, $\mathrm{G}^{\dagger}$ is the pseudo inverse of G. Figure.2 illustrates the block diagram implementation of (21), where the control input is filtered i_m^{filt} so as not to excite any of the system's flexible modes in order to use (20) which is only valid in the system's low frequency range. Then the estimated rigid motion is used to estimate system's parameters through (21).

B. States Estimation

According to (18), there exist (n) flexible modes that can be excited by the unfiltered control input i_m^{init}. It is important to emphasize that the control input is filtered just to determine system parameters by performing a rigid motion maneuver that allows using (20) and (21). On the other hand, the control input can excite any of the system flexible modes of (18). Therefore, position of each lumped mass has to be determined. Rewriting (17) and replacing the actual parameters with the estimated ones we obtain the following differential equation

$$\frac{d\theta_1(t)}{dt} + \frac{\widehat{k}}{\widehat{B}}\theta_1(t) = \beta(t) \tag{22}$$

$$\beta(t) \triangleq \frac{\widehat{B}\ \dot{\theta}_m(t) + \widehat{k}\ \theta_m(t) - \widehat{\tau_{reac}}(t)}{\widehat{B}}$$

then it can be shown that estimate of the first lumped mass is

$$\widehat{\theta}_1(t) = c_6 e^{-\frac{\widehat{B}}{\widehat{k}}t} + \int_0^T \beta(\tau)\ e^{\frac{\widehat{B}}{\widehat{k}}(t-\tau)}d\tau \tag{23}$$

Similarly, estimate of the second and third lumped masses can be obtained through the following equation

$$\widehat{\theta}_2(t) = c_7 e^{-\frac{\widehat{B}}{\widehat{k}}t} + \int_0^T \Lambda(\tau) e^{\frac{\widehat{k}}{\widehat{B}}(t-\tau)}d\tau \tag{24}$$

$$\Lambda(\tau) \triangleq \frac{J_1\widehat{\ddot{\theta}}_1 - \widehat{B}(\dot{\theta}_0 - \widehat{\dot{\theta}}_1) - \widehat{k}(\theta_0 - \theta_1) + \widehat{B}\ \widehat{\dot{\theta}}_1 + \widehat{k}\ \widehat{\theta}_1}{\widehat{B}}$$

$$\widehat{\theta}_3(t) = c_8 e^{-\frac{\widehat{B}}{\widehat{k}}t} + \int_0^T \varepsilon(\tau)\ e^{\frac{\widehat{k}}{\widehat{B}}(t-\tau)}d\tau \tag{25}$$

$$\varepsilon(\tau) \triangleq \frac{J_2\ \widehat{\ddot{\theta}}_2 - \widehat{B}(\widehat{\dot{\theta}}_1 - \widehat{\dot{\theta}}_2) - \widehat{k}(\widehat{\theta}_1 - \widehat{\theta}_2) + \widehat{B}\ \widehat{\dot{\theta}}_2 + \widehat{k}\ \widehat{\theta}_2\ .}{\widehat{B}}$$

In general the position of the i^{th} lumped mass can be obtained through the following recursive formula

$$\widehat{\theta}_i(t) = c_i e^{-\frac{\widehat{B}}{\widehat{k}}t} + \int_0^T \Omega(\tau)\ e^{\frac{\widehat{k}}{\widehat{B}}(t-\tau)}d\tau \tag{26}$$

$$\Omega(\tau) \triangleq \frac{g(J_{i-1}, \widehat{\theta}_{i-1}, \widehat{\dot{\theta}}_{i-1}, \widehat{\ddot{\theta}}_{i-1}, \widehat{k}, \widehat{B})}{\widehat{B}}$$

IV. Experimental Results

TABLE I
EXPERIMENTAL PARAMETERS

Parameter	Value	Parameter	Value
J_1	5152.99 gcm^2	g_{dist}	100 rad/sec
J_2	5152.99 gcm^2	g_{lpf}	100 rad/sec
J_3	6192.707 gcm^2	f_{init}	1 rad/sec
J_m	209 gcm^2	k_{act}	1.627 KN/m
k_b	235 rpm/v	k_t	40.6 mNm/A
8	Maxon-Ec-Motor 229427	1,6,7	Inertial loads
4	Spring-$k_{th} = 1.62kN/m$	2,3,5	Optical encoders

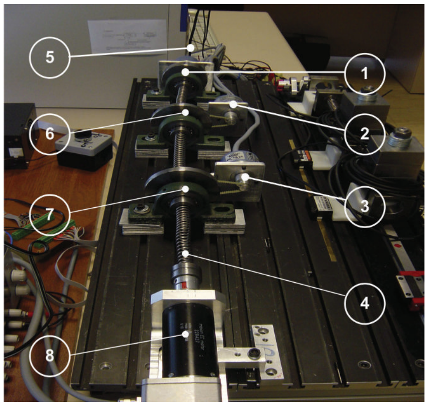
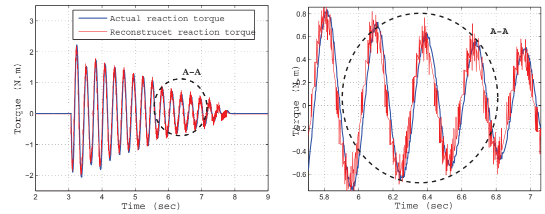


Fig. 3. Experimental setup.

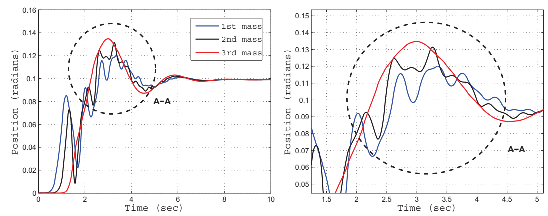
TABLE II
PARAMETER IDENTIFICATION EXPERIMENTAL RESULTS

Experiment	$\hat{k}$ (kN/m)	$\widehat{B}$ (Nsec/m)
1st Exp	1.5796	0.0888
2nd Exp	1.5336	0.0878
3rd Exp	1.6459	0.0887
4rd Exp	1.5116	0.0889
5rd Exp	1.5625	0.0893



(a) $\widehat{\tau_{reac}}(t)$ vs. reconstructed $\widehat{\tau_{reac}}(t)$ (b) Magnified plot

Fig. 4. Reaction torque and reconstructed reaction torque through estimated parameters



(a) Masses actual positions (b) Magnified plot

Fig. 5. Flexible response of the 3 DOF flexible system

In order to verify the validity of the proposed parameter identification and states estimation technique, experiments are performed on an inertial lumped flexible system with three degrees of freedom as depicted in Fig.3. Actuator current and velocity are measured while an encoder is attached to each lumped mass in order to verify the validity of the recursive equations (26) by comparing the actual measurements taken by the encoders with the estimated ones determined through (26). In the following two experiments only two measurements are taken from the actuator, namely actuator's current and velocity. The plant is kept free from any attached sensors. However, the natural feedback caused by the instantaneous reaction is considered as an alternative to actual measurement taken by attached sensors.

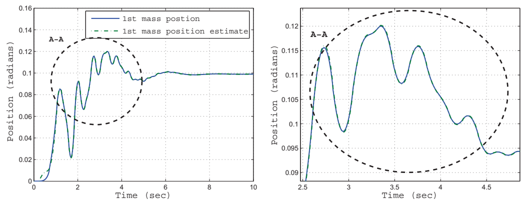
A. Parameter Identification Experiment

The system parameter identification is conducted by performing any arbitrary rigid maneuver to guarantee that (20) can be used then system parameters are estimated through (21). The entire experiment depends on two measurement from the actuator while the flexible multi-degree-of freedom system is kept free from any measurement. Table.II summarizes the parameter identification results, where the rigid maneuver was performed 5 times and the corresponding viscous damping coefficient and stiffness are identified. Consequently, the average viscus damping and stiffness are $1.54653\ kN/m$ and $0.08433\ Nsec/m$, respectively. The estimated damping coefficient and stiffness are then used to reconstruct the reaction torque signal so as to compare this signal with the estimated reaction torque as depicted in Fig.4. The previous figure demonstrates that the estimated parameters are close to the actual ones. However, the difference between the actual known before hand parameters and estimated ones is less then 5 percent. In addition, the noisy nature of the reconstructed signal shown in Fig.4 is due to the direct differentiation of the position signal. Nevertheless, it doesn't affect any further computation as it is only computed to illustrate that the difference between estimated and actual parameters can be negligible.

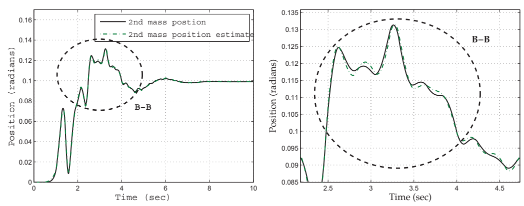
B. States estimation experimental results

Unlike the previous experiment that requires flexible system to perform an arbitrary rigid maneuver, the states estimation experiment can be performed anywhere along the system's entire frequency range. In other words, for the parameter estimation experiment the control input has to be filtered so as not to excite the system flexible modes of (18) that is not the case in this experiment as (20) has to be verified under any arbitrary control input regardless to its energy content. Figure 5 illustrates a flexible behavior of the system when the control input contains energy at system's flexible modes. In this case, Equation (20) is no longer valid and the recursive equation (26) has to be used to recursively estimate position of each lumped mass along the flexible system.

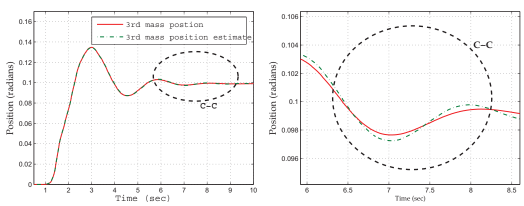
First, system parameters $\widehat{B}$ and $\widehat{k}$ are identified then used along with the reaction torque $\widehat{\tau_{reac}}(t)$ and actuator's velocity to observer the angular position of each lumped mass of the flexible system through (26). Optical encoders are attached to each lumped mass as shown in Fig.3 in order to compare the



(a) Actual and estimated 1^{st} mass position (b) Magnified Plot A-A



(c) Actual and estimated 2^{nd} mass position (d) Magnified Plot B-B



(e) Actual and estimated 3^{rd} mass position (f) Magnified Plot C-C

Fig. 6. Position estimation experimental results

actual measured position with the observed ones. Estimation of each lumped mass is determined and compared with the actual encoder measurements as illustrated in Fig.6. The results illustrated in Fig.6 demonstrate the validity of (26) and the possibility of practical implementation of the proposed technique.

V. Conclusion

The problem of keeping flexible systems free from any measurement while considering the actuator as a single platform for measurement is addressed in this work. Disturbance, flexibility and the Newtonian Action-Reaction principle are combined to formulate a framework which allows identifying system parameters and observing system states through measurements taken from the actuator side. The flexible system's reaction due to an action imposed by the actuator is investigated. Moreover, a model based mathematical representation of the reaction signal is derived for a simple system with few flexible modes and for an infinite modes system. It turns out that reaction signal carries sufficient coupled information about the flexible system such as system parameter, dynamics and externally applied torques or forces. Furthermore, the entire coupled signal denoted as the incident reaction torque or force is determined or estimated from the interface point of the actuator with the flexible plant using actuator's current and velocity. Then system parameters and dynamics are decoupled out of the reaction torque. The experimental results demonstrate the validity of the proposed technique where the difference between the identified parameters and the actual known before hand ones is less than five percent. In addition, on-line comparison of the observed positions with the actual measurements demonstrates the possibility of keeping these flexible systems free from any attached sensors during a motion and vibration control assignment. Furthermore, the obtained results encourage the attempt of extending this work for the more practical infinite modes systems such as flexible beams and flexible robot manipulators.

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