Autonomous Systems

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Outline

- Introduction
- Motion control of mobile robots
- Observability of nonlinear systems

Figure: PackBot: a military mobile robot.

Consider the following mobile robot:

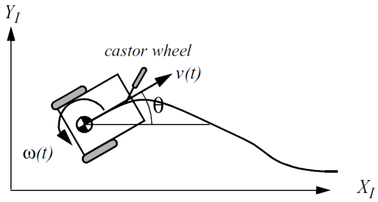

Figure: Mobile robot moving along a trajectory.

$v(t)$ and $\omega(t)$ denote the linear and angular velocities of the robot, respectively.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}, \quad (1)$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega, \quad (2)$$

$$\dot{\mathbf{x}} = f(\mathbf{x})v + g(\mathbf{x})\omega, \quad (3)$$

where

$$f(\mathbf{x}) \triangleq \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{bmatrix} \text{ and } g(\mathbf{x}) \triangleq \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Motion Control of Mobile Robots

Our goal is to move the mobile robot to a reference position in a finite time using the control inputs v and ω

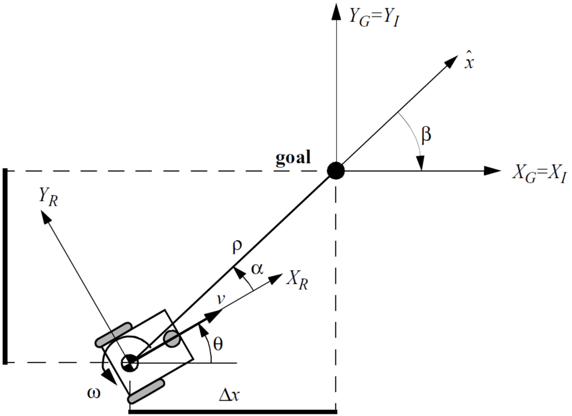

Figure: Motion control of a mobile robots towards a reference position.

Our mobile robot have the following representation using polar coordinates:

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -\cos\alpha & 0 \\ \frac{1}{\rho}\sin\alpha & -1 \\ -\frac{1}{\rho}\sin\alpha & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}. \tag{4}$$

The coordinate transformation is given by

$$\rho = \sqrt{\Delta x^2 + \Delta y^2}, \tag{5}$$

$$\alpha = -\theta + \arctan 2(\Delta y^2, \Delta x^2), \tag{6}$$

$$\beta = -\theta - \alpha. \tag{7}$$

Setting $v = -v$ and $\omega = -\omega$ yields

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} \cos\alpha & 0 \\ -\frac{1}{\rho}\sin\alpha & 1 \\ \frac{1}{\rho}\sin\alpha & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} \tag{8}$$

Now consider the following control law

$$v = k_\rho \rho \textit{ and } \omega = k_\alpha \alpha + k_\beta \beta. \tag{9}$$

Substituting (9) in (8) yields

$$\begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -k_\rho \rho \cos \alpha \\ k_\rho \sin \alpha - k_\alpha \alpha - k_\beta \beta \\ -k_\rho \sin \alpha \end{bmatrix}. \tag{10}$$

Now lets check the stability within the vicinity of the equilibrium point/points.

- $-k_\rho \sin \alpha = 0 \implies \sin \alpha = 0 \implies \alpha = 0$
- $k_\rho \sin \alpha - k_\alpha \alpha - k_\beta \beta = 0 \implies \beta = 0$
- $-k_\rho \rho \cos \alpha = 0 \implies \rho = 0.$

Therefore, the system has a unique equilibrium point at $(0, 0, 0)$.

Now lets analyze the stability within the vicinity of the equilibrium point. The Jacobian matrix (**A**) is given by

$$\mathbf{A} = \begin{bmatrix} \frac{\partial f_1}{\partial \rho} & \frac{\partial f_1}{\partial \alpha} & \frac{\partial f_1}{\partial \beta} \\ \frac{\partial f_2}{\partial \rho} & \frac{\partial f_2}{\partial \alpha} & \frac{\partial f_2}{\partial \beta} \\ \frac{\partial f_3}{\partial \rho} & \frac{\partial f_3}{\partial \alpha} & \frac{\partial f_3}{\partial \beta} \end{bmatrix} = \begin{bmatrix} -k_\rho & 0 & 0 \\ 0 & -(k_\rho - k_\alpha) & -k_\beta \\ 0 & -k_\rho & 0 \end{bmatrix}. \tag{11}$$

The characteristic equation is given by

$$\mid \lambda \mathbf{I} - \mathbf{A} \mid = 0. \tag{12}$$

Therefore, the equilibrium point is stable if $k_\rho > 0$, $-k_\beta > 0$ and $k_\alpha - k_\rho > 0$.

Observability of Nonlinear Systems

Consider a nonlinear system with the following dynamics:

$$\dot{\mathbf{x}} = f(\mathbf{x}) \text{ and } \mathbf{y} = h(\mathbf{x}). \tag{13}$$

This system is observable of the matrix $\mathcal{O} = dG$ has a full rank, where G is given by

$$G = \begin{bmatrix} h \\ L_f(h) \\ \vdots \\ L_f^{n-1}(h) \end{bmatrix}. \tag{14}$$

Analyze the observability of the following system:

$$\dot{x}_1 = \frac{1}{2}x_1^2 + \exp(x_2) + x_2 \quad (15)$$

$$\dot{x}_2 = x_1^2 \quad (16)$$

$$y = x_1 \quad (17)$$

$$G = \begin{bmatrix} h \\ L_f h \end{bmatrix} = \begin{bmatrix} x_1 \\ \frac{1}{2}x_1^2 + \exp(x_2) + x_2 \end{bmatrix} \quad (18)$$

$$\mathcal{O} = \begin{bmatrix} 1 & 0 \\ x_1 & \exp(x_2) + 1 \end{bmatrix}. \quad (19)$$

The matrix $\mathcal{O}$ has rank of 2.

Thanks

Questions please