Medical Robotics

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Basic components of a bilateral control system are: An operator, master device, slave device, and an environment.

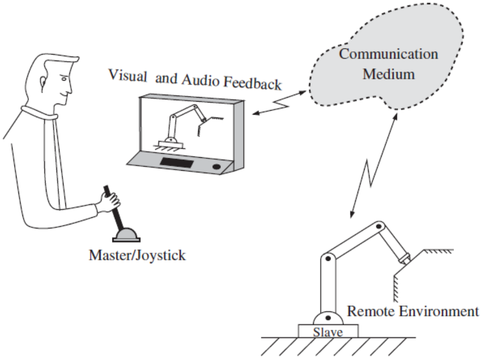

Figure: Bilateral control (Hokayem and Spong).

Bilateral Control Without Scaling

Basic components of a bilateral control system are: An operator, master device, slave device, and an environment.

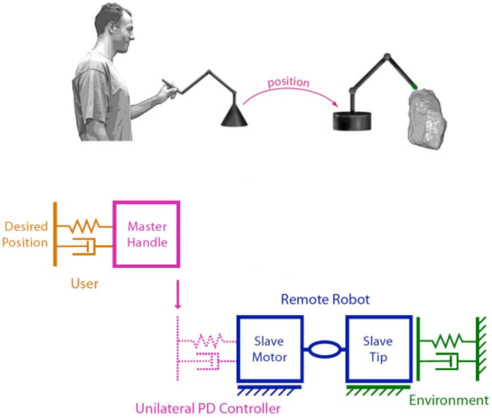

Figure: Bilateral control.

Bilateral Control Without Scaling

Equation of motion of the master robot is given by

$$a_m(q_m)\ddot{q}_m + b(q_m, \dot{q}_m) + g(q_m) = \tau_m - \tau_{mext} - \tau_h, \qquad (1)$$

where $a_m(q_m)$ is the inertia matrix of the master. Further, q_m is the generalized coordinate of the master. $b(q_m, \dot{q}_m)$ and $g(q_m)$ are the nonlinear friction and gravitational terms, respectively. Furthermore, τ_m, τ_{mext}, and τ_h are the torque of the master, the external torque on the master, and the input torque of the operator, respectively. Equation of motion of the slave device is given by

$$a_s(q_s)\ddot{q}_s + b(q_s, \dot{q}_s) + g(q_s) = \tau_s - \tau_{sext} - \tau_s. \qquad (2)$$

In (4), q_s is the generalized coordinate of the slave robot.

In (2), $a_s(q_s)$ is the inertia matrix of the slave robot. Further, $b(q_s, \dot{q}_s)$ and $g(q_s)$ are the nonlinear friction and gravitational terms, respectively. Furthermore, τ_s, τ_{sext}, and τ_s are the torque of the slave, the external torque on the slave, and the input torque of the environment, respectively.

The interaction between the operator and the master robot is given by

$$a_h(q_h)\ddot{q}_h + b(q_s, \dot{q}_h) + g(q_h) = \tau_{op} - \tau_h, \tag{3}$$

$$\tau_h = D_h(\dot{q}_m - \dot{q}_h) + k_h(q_m - q_h). \tag{4}$$

Further, the interaction between the slave robot and the environment is modeled using

$$\tau_e = D_e(\dot{q}_s - \dot{q}_e) + k_e(q_s - q_e). \tag{5}$$

Bilateral Control Without Scaling

General relationship between position and forces in bilateral control can be described using the hybrid matrix **H**

$$\begin{bmatrix} f_h \\ q_s \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} q_m \\ f_s \end{bmatrix} = \mathbf{H} \begin{bmatrix} q_m \\ f_s \end{bmatrix}. \tag{6}$$

h_{12} refers to the accuracy of force reflection (the force tracking), whereas h_{21} refers to the position tracking of the slave with respect to the master, h_{11} denotes the master-side mechanical impedance, and h_{22} can be interpreted as a slave-side mechanical admittance.

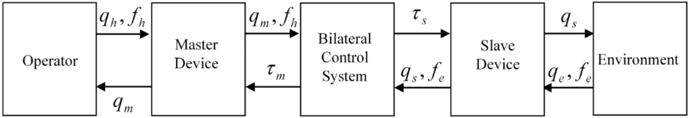

Figure: Bilateral control structure.

Bilateral Control Without Scaling

The goal of bilateral control is to achieve "transparent teleoperatation": the operator feels as if he is manipulating the remote environment directly. The bilateral control desired (ideal) conditions can be represented in the following matrix

$$\begin{bmatrix} f_h \\ q_s \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} q_m \\ f_s \end{bmatrix}. \tag{7}$$

Assume that disturbance compensation is applied at the master and slave robots. Therefore, their dynamics are given by

$$a_m \ddot{q}_m = a_{mn} \ddot{q}_m^{des} - \Delta P_m, \tag{8}$$

where a_{mn} is the nominal inertia scalar and ΔP_m is the deviation mismatch between the estimated and actual disturbance.

$$a_s \ddot{q}_s = a_{sn} \ddot{q}_s^{des} - \Delta P_s, \tag{9}$$

where a_{sn} is the nominal inertia scalar and ΔP_s is the deviation mismatch between the estimated and actual disturbance. These systems are realized using the following control inputs:

$$\tau_m = \widehat{\tau}_{mdist} + a_{mn} \ddot{q}_m^{des}, \tag{10}$$

$$\tau_s = \widehat{\tau}_{sdist} + a_{sn} \ddot{q}_s^{des}, \tag{11}$$

where $\ddot{q}_m^{des}$ and $\ddot{q}_s^{des}$ are the desired acceleration of the master and slave robots, respectively.

Bilateral Control Design

The second derivative of position error is given by

$$\ddot{e}_{qb} = \ddot{q}_m - \ddot{q}_s = \ddot{q}_m^{des} - \ddot{q}_s^{des} - \left(\frac{\Delta P_m}{a_{mn}} - \frac{\Delta P_s}{a_{sn}} \right), \tag{12}$$

$$\ddot{q}_{qb}^{des} = \ddot{q}_m^{des} - \ddot{q}_s^{des}, \tag{13}$$

$$P_{qb} = \left(\frac{\Delta P_m}{a_{mn}} - \frac{\Delta P_s}{a_{sn}} \right), \tag{14}$$

Therefore, (12) is given by

$$\ddot{e}_{qb} = \ddot{q}_{qb}^{des} - P_{qb}, \tag{15}$$

Based on (15), we devise the following desired acceleration:

$$\ddot{q}_{qb}^{des} = \widehat{P}_{qb} - k_D \dot{e}_{qb} - k_P e_{qb}. \tag{16}$$

Bilateral Control Design

From a control theoretic point of view the main goals of teleoperation are twofold:

- Stability: Maintain stability of the closed-loop system irrespective of the behavior of the operator or the environment.
- Telepresence: Provide the human operator with a sense of telepresence, with the latter regarded as transparency of the system between the environment and the operator.

These tasks are generally conflicting. However, satisfying these requirements extends the capabilities of the human by scaling her/his power into manipulating huge objects, as in outer space construction, or performing delicate tasks, as in microsurgery; thus projecting his/her expertise into distant locations.

Bilateral Control Design

Mathematically, we think of a teleoperator as two robotic subsystems a master and a slave that exchange signals (positions, velocities and/or forces); in which the slave tries to mimic the behavior of the master which in turn takes into account the input torques from the slave. A linear model of master/slave system can be written as

$$\mathbf{M}_{\mathrm{m}}\ddot{\mathbf{x}}_{\mathrm{m}} + \mathbf{B}_{\mathrm{m}}\dot{\mathbf{x}}_{\mathrm{m}} = f_{\mathrm{m}} + f_{\mathrm{h}} \quad (17)$$

$$\mathbf{M}_{\mathrm{s}}\ddot{\mathbf{x}}_{\mathrm{s}} + \mathbf{B}_{\mathrm{s}}\dot{\mathbf{x}}_{\mathrm{s}} = f_{\mathrm{s}} - f_{\mathrm{e}} \quad (18)$$

where f_{h} and f_{e} are the external forces exerted by the human operator and the environment, respectively. A more detailed nonlinear model can be written using Lagranges equations as

$$\mathbf{M}_{\mathrm{m}}\ddot{\mathbf{x}}_{\mathrm{m}} + \mathbf{C}_{\mathrm{m}}(\mathbf{x}_{\mathrm{m}}, \dot{\mathbf{x}}_{\mathrm{m}})\dot{\mathbf{x}}_{\mathrm{m}} = f_{\mathrm{m}} + f_{\mathrm{h}} \quad (19)$$

$$\mathbf{M}_{\mathrm{s}}\ddot{\mathbf{x}}_{\mathrm{s}} + \mathbf{C}_{\mathrm{s}}\dot{\mathbf{x}}_{\mathrm{s}} = f_{\mathrm{s}} - f_{\mathrm{e}} \quad (20)$$

We use the following properties: $\mathbf{M} = \mathbf{M}^{\mathrm{T}}$ (positive definite) and $\dot{\mathbf{M}} = -2\mathbf{C}$ (skew symmetric).

A dynamical system given by

$$\dot{x} = f(x, u) \tag{21}$$
$$y = h(x, u) \tag{22}$$

is said to be passive if there exists a continuously differentiable semidefinite scalar function $V(x)$ (storage function) such that

$$\dot{V} \leq u^{\mathrm{T}} y \quad \left(\equiv \int_0^t u(\eta)^{\mathrm{T}} y(\eta) \mathrm{d}\eta \geq V(t) - V(0) \right) \tag{23}$$

and lossless if

$$\dot{V} \leq u^{\mathrm{T}} y \quad \left(\equiv \int_0^t u(\eta)^{\mathrm{T}} y(\eta) \mathrm{d}\eta = V(t) - V(0) \right) \tag{24}$$

A passive system has a stable origin and we can think of V as the system's energy.

Bilateral Control Design

Given the properties (PD) and (SS), and assuming that the human and environment are passive, i.e., $\int_0^t \left(f_h^{\mathrm{T}}\dot{x}_m - f_e^{\mathrm{T}}\dot{x}_s\right) \mathrm{d}\eta \geq 0$, then the nonlinear system (NL) with inputs $[f_m, f_s]$ and outputs $[\dot{x}_m, \dot{x}_s]$ is passive with respect to the storage function

$$V = \frac{1}{2}\begin{bmatrix}\dot{x}_{\mathrm{m}}\\ \dot{x}_{\mathrm{s}}\end{bmatrix}\begin{bmatrix}M_{\mathfrak{m}} & 0\\ 0 & M_{\mathrm{s}}\end{bmatrix}\begin{bmatrix}\dot{x}_{\mathrm{m}}\\ \dot{x}_{\mathrm{s}}\end{bmatrix} \tag{25}$$

Therefore, we can look at the system (NL) as a lossless system by taking forces as inputs and velocities as outputs, and studying energy exchange occurring (a) within the teleoperator and (b) with the external world, i.e. human and remote environment. This is particularly useful since we know that a series cascade of passive two-ports is passive (with respect to the remaining open ports), and a series cascade of a two-port with a one-port network is also passive (with respect to the remaining open port).

Example:

Consider the system

$$\dot{x}_1 = x_2 \tag{26}$$

$$\dot{x}_2 = -x_1^3 + u \tag{27}$$

Let

$$V = \frac{x_1^4}{4} + \frac{x_1^2}{2} \tag{28}$$

This implies that

$$\dot{V} = x_1^3 x_2 - x_2 x_1^3 + x_2 u = x_2 u \tag{29}$$

Choose $y = x_2$ and we see that we get a passive system which can be stabilized with the control $u = -kx_2$.

Passivation of m-link Robot:

Consider the following equations of motion for a typical kinematic system,

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + D\dot{q} + g(q) = u \tag{30}$$

Consider the tracking problem that seeks to make $e = q - q_{\mathrm{r}}$ small. The tracking error dynamics is given by

$$M(q)\ddot{e} + C(q,\dot{q})\dot{e} + D\dot{e} + g(q) = u \tag{31}$$

We want to stabilize around $\dot{e} = 0$ and $e = 0$. But this is not necessarily a stable equilibrium point. Let

$$u = g(q) - k_p e + v \tag{32}$$

where k_p is a positive definite symmetric matrix and v is an additional control input. We get

$$M(q)\ddot{e} + C(q,\dot{q})\dot{e} + D\dot{e} + k_p e = v \tag{33}$$

Passivation of m-link Robot:

We now consider

$$V = \frac{1}{2}\dot{e}^{\mathrm{T}} M(q) e + \frac{1}{2} e^{\mathrm{T}} k_p e \tag{34}$$

as a storage function, then we obtain

$$\begin{aligned} \dot{V} &= \dot{e}^{\mathrm{T}} M \ddot{e} + \frac{1}{2}\dot{e}^{\mathrm{T}} \dot{M} \dot{e} + e^{\mathrm{T}} k_p \dot{e} & (35) \\ &= \frac{1}{2}\dot{e}^{\mathrm{T}} (\dot{M} - 2C)\dot{e} - \dot{e}^{\mathrm{T}} D \dot{e} - \dot{e}^{\mathrm{T}} k_p e + \dot{e}^{\mathrm{T}} v + e^{T} k_p e & (36) \\ &= \leq \dot{e}^{\mathrm{T}} v & (37) \end{aligned}$$

Defining the output as $y = \dot{e}$, we see that the system with input v and output y is passive with V as a storage function.
The role of the passifying feedback $g(q) - k_p e$ is to reshape the potential energy to $\frac{1}{2} e^{\mathrm{T}} k_p e$ which has a unique minimum at $e = 0$.

Thanks

Questions please