Autonomous Systems

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April 2, 2017

Vector Relative Degree

Consider a nonlinear MIMO system of the form

$$\dot{\mathbf{x}} = f(\mathbf{x}) + \sum_{i=1}^{m} u_i g_i(\mathbf{x}) = f(\mathbf{x}) + G(\mathbf{x})\mathbf{u}, \tag{1}$$

$$y_1 = h_1(\mathbf{x}) \tag{2}$$

$$\vdots \tag{3}$$

$$y_m = h_m(\mathbf{x}) \tag{4}$$

The MIMO system has a vector relative degree $(\rho_1, \rho_2, \ldots, \rho_m)$ if the following matrix is nonsingular:

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} \mathcal{L}_{g_1}\mathcal{L}_f^{\rho_1-1}h_1(\mathbf{x}) & \ldots & \mathcal{L}_{g_m}\mathcal{L}_f^{\rho_1-1}h_1(\mathbf{x}) \\ \mathcal{L}_{g_1}\mathcal{L}_f^{\rho_2-1}h_2(\mathbf{x}) & \ldots & \mathcal{L}_{g_m}\mathcal{L}_f^{\rho_2-1}h_2(\mathbf{x}) \\ \vdots & \ldots & \vdots \\ \mathcal{L}_{g_1}\mathcal{L}_f^{\rho_m-1}h_m(\mathbf{x}) & \ldots & \mathcal{L}_{g_m}\mathcal{L}_f^{\rho_m-1}h_m(\mathbf{x}) \end{bmatrix}, \tag{5}$$

Vector Relative Degree

Consider the following mobile robot:

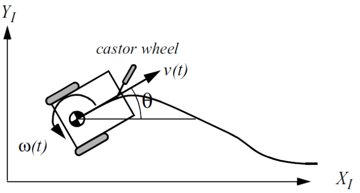

Figure: Mobile robot moving along a trajectory.

$v(t)$ and $\omega(t)$ denote the linear and angular velocities of the robot, respectively.

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) & 0 \\ \sin(\theta) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}, \qquad (6)$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{bmatrix} v + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \omega, \qquad (7)$$

$$\dot{\mathbf{x}} = g_1(\mathbf{x})v + g_2(\mathbf{x})\omega, \qquad (8)$$

where

$$g_1(\mathbf{x}) \triangleq \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \\ 0 \end{bmatrix} \text{ and } g_2(\mathbf{x}) \triangleq \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

The system can be represented in the form

$$\dot{\mathbf{x}} = f(\mathbf{x}) + \sum_{i=1}^{m} u_i g_i(\mathbf{x}) = 0 + g_1(\mathbf{x})u_1 + g_2(\mathbf{x})u_2, \tag{9}$$

For trajectory tracking problem, the natural outputs of the system are

$$y_1 = x \quad \text{and} \quad y_2 = y, \tag{10}$$

Therefore,

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} \mathcal{L}_{g_1}\mathcal{L}_f^{\rho_1-1}h_1(\mathbf{x}) & \mathcal{L}_{g_2}\mathcal{L}_f^{\rho_1-1}h_1(\mathbf{x}) \\ \mathcal{L}_{g_1}\mathcal{L}_f^{\rho_2-1}h_2(\mathbf{x}) & \mathcal{L}_{g_2}\mathcal{L}_f^{\rho_2-1}h_2(\mathbf{x}) \end{bmatrix}, \tag{11}$$

Let us assume that the relative degree of the system is $(\rho_1, \rho_2) = (1, 1)$.

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} \mathcal{L}_{g_1}\mathcal{L}_f^{0}h_1(\mathbf{x}) & \mathcal{L}_{g_2}\mathcal{L}_f^{0}h_1(\mathbf{x}) \\ \mathcal{L}_{g_1}\mathcal{L}_f^{0}h_2(\mathbf{x}) & \mathcal{L}_{g_2}\mathcal{L}_f^{0}h_2(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} \mathcal{L}_{g_1}h_1(\mathbf{x}) & \mathcal{L}_{g_2}h_1(\mathbf{x}) \\ \mathcal{L}_{g_1}h_2(\mathbf{x}) & \mathcal{L}_{g_2}h_2(\mathbf{x}) \end{bmatrix}, \tag{12}$$

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} \mathcal{L}_{g_1} h_1(\mathbf{x}) & \mathcal{L}_{g_2} h_1(\mathbf{x}) \\ \mathcal{L}_{g_1} h_2(\mathbf{x}) & \mathcal{L}_{g_2} h_2(\mathbf{x}) \end{bmatrix}, \tag{13}$$

$$\mathcal{L}_{g_1} h_1(\mathbf{x}) = \frac{\partial h_1}{\partial x} g_1(\mathbf{x}) = \cos\theta \tag{14}$$

$$\mathcal{L}_{g_2} h_1(\mathbf{x}) = \frac{\partial h_1}{\partial x} g_2(\mathbf{x}) = 0 \tag{15}$$

$$\mathcal{L}_{g_1} h_2(\mathbf{x}) = \frac{\partial h_2}{\partial x} g_1(\mathbf{x}) = \sin\theta \tag{16}$$

$$\mathcal{L}_{g_2} h_2(\mathbf{x}) = \frac{\partial h_2}{\partial x} g_2(\mathbf{x}) = 0 \tag{17}$$

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{bmatrix}, \tag{18}$$

We conclude that $\mathbf{A}(\mathbf{x})$ is singular and the relative degree is not $(1, 1)$. The problem is that the input u_1 or v appears in the derivative of both outputs, while the input u_2 (or ω) does not.

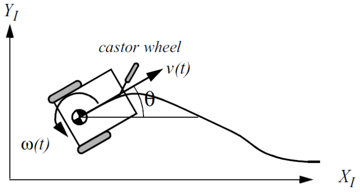

Figure: Mobile robot.

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \end{bmatrix}, \qquad (19)$$

v appears in the derivative of both outputs.
ω does not appear in the derivative of any output.

Let us try to make v appear later in a higher-order derivative of the output.

$$v = \zeta \qquad (20)$$
$$\dot{\zeta} = \tau \qquad (21)$$

The new representation of the system is

$$\dot{x} = \zeta\cos\theta \qquad (22)$$
$$\dot{y} = \zeta\sin\theta \qquad (23)$$
$$\dot{\zeta} = \tau \qquad (24)$$
$$\dot{\theta} = \omega \qquad (25)$$

Vector Relative Degree

The new system with the extended state is

$$\dot{x} = \zeta \cos\theta \quad (26)$$

$$\dot{y} = \zeta \sin\theta \quad (27)$$

$$\dot{\zeta} = \tau \quad (28)$$

$$\dot{\theta} = \omega \quad (29)$$

In a compact form

$$\dot{\mathbf{x}} = f(\mathbf{x}) + g_1(\mathbf{x})\tau + g_2(\mathbf{x})\omega \quad (30)$$

where

$$\mathbf{x} = \begin{bmatrix} x \\ y \\ \zeta \\ \theta \end{bmatrix}, \; f(\mathbf{x}) = \begin{bmatrix} \zeta\cos\theta \\ \zeta\sin\theta \\ 0 \\ 0 \end{bmatrix}, \; g_1(\mathbf{x}) = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \; g_2(\mathbf{x}) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad (31)$$

Vector Relative Degree

$$\mathcal{L}_{g_1} h_1(\mathbf{x}) = \frac{\partial h_1}{\partial x} g_1(\mathbf{x}) = 0 \quad (32)$$

$$\mathcal{L}_{g_2} h_1(\mathbf{x}) = \frac{\partial h_1}{\partial x} g_2(\mathbf{x}) = 0 \quad (33)$$

$$\mathcal{L}_{g_1} h_2(\mathbf{x}) = \frac{\partial h_2}{\partial x} g_1(\mathbf{x}) = 0 \quad (34)$$

$$\mathcal{L}_{g_2} h_2(\mathbf{x}) = \frac{\partial h_2}{\partial x} g_2(\mathbf{x}) = 0 \quad (35)$$

The inputs do not appear in the first order derivative of the outputs. Let us see the second order derivatives

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} \mathcal{L}_{g_1}\mathcal{L}_f h_1(\mathbf{x}) & \mathcal{L}_{g_2}\mathcal{L}_f h_1(\mathbf{x}) \\ \mathcal{L}_{g_1}\mathcal{L}_f h_2(\mathbf{x}) & \mathcal{L}_{g_2}\mathcal{L}_f h_2(\mathbf{x}) \end{bmatrix}, \quad (36)$$

First we have to calculate $\mathcal{L}_f h_1(\mathbf{x})$ and $\mathcal{L}_f h_2(\mathbf{x})$

Vector Relative Degree

$$\mathcal{L}_f h_1(\mathbf{x}) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \zeta \cos\theta \\ \zeta \sin\theta \\ 0 \\ 0 \end{bmatrix} = \zeta \cos\theta \qquad (37)$$

$$\mathcal{L}_f h_2(\mathbf{x}) = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \zeta \cos\theta \\ \zeta \sin\theta \\ 0 \\ 0 \end{bmatrix} = \zeta \sin\theta \qquad (38)$$

Now we calculate the entries of $\mathbf{A}(\mathbf{x})$

$$\mathcal{L}_{g_1} \mathcal{L}_f h_1(\mathbf{x}) = \begin{bmatrix} 0 & 0 & \cos\theta & -\zeta \sin\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \cos\theta \qquad (39)$$

Vector Relative Degree

$$\mathcal{L}_{g_2}\mathcal{L}_f h_1(\mathbf{x}) = \begin{bmatrix} 0 & 0 & \cos\theta & -\zeta\sin\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = -\zeta\sin\theta \qquad (40)$$

$$\mathcal{L}_{g_1}\mathcal{L}_f h_2(\mathbf{x}) = \begin{bmatrix} 0 & 0 & \sin\theta & \zeta\cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \sin\theta \qquad (41)$$

$$\mathcal{L}_{g_2}\mathcal{L}_f h_2(\mathbf{x}) = \begin{bmatrix} 0 & 0 & \sin\theta & \zeta\cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \zeta\cos\theta \qquad (42)$$

Therefore $\mathbf{A}(\mathbf{x})$ is given by

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} \cos\theta & -\zeta\sin\theta \\ \sin\theta & \zeta\cos\theta \end{bmatrix} \quad , \quad |\mathbf{A}(\mathbf{x})| = \zeta \tag{43}$$

As long as $\zeta \neq 0$, the matrix $\mathbf{A}$ is nonsingular and the relative degree of the system is $(2, 2)$. We also see that $\rho = \rho_1 + \rho_2 = 4 = n$, where n is the dimension of the extended system. Now the control system design becomes relatively straightforward. We device the following control law:

$$\begin{bmatrix} \tau \\ \omega \end{bmatrix} = \mathbf{A}(\mathbf{x})^{-1} \begin{bmatrix} -\ddot{x}_{\mathrm{d}} - k_1\dot{e}_x - k_2 e_x \\ -\ddot{y}_{\mathrm{d}} - k_3\dot{e}_y - k_4 e_y \end{bmatrix} \tag{44}$$

where the position error along x- and y-axis are given by

$$e_x = x - x_{\mathrm{d}} \quad \text{and} \quad e_y = y - y_{\mathrm{d}} \tag{45}$$

Furthermore, x_{d} and y_{d}, $\dot{x}_{\mathrm{d}}$ and $\dot{y}_{\mathrm{d}}$, $\ddot{x}_{\mathrm{d}}$ and $\ddot{y}_{\mathrm{d}}$ are the desired position, velocity, and acceleration along x- and y-axis, respectively.

Vector Relative Degree

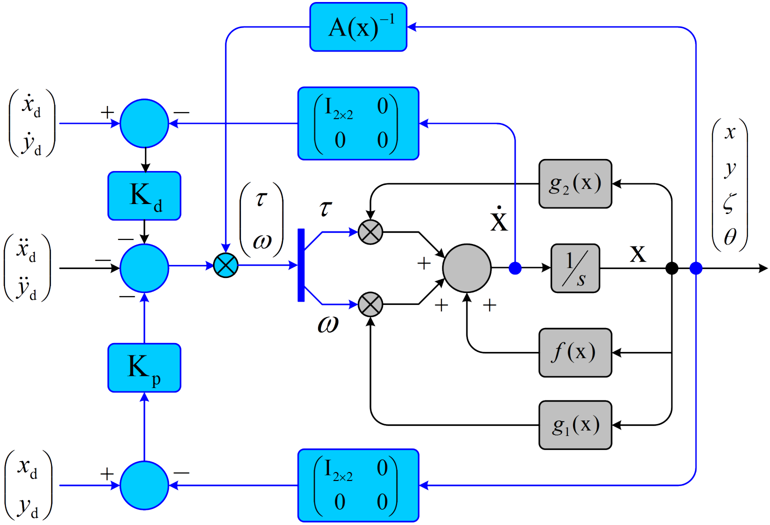

Figure: Control of a tricycle mobile robot via dynamic state feedback.

Now consider that the control inputs are the spinning velocities of the first and second wheels $\dot{\varphi}_1$ and $\dot{\varphi}_2$, respectively.

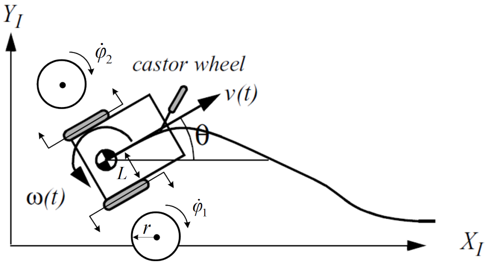

Figure: Mobile robot moving in a global reference frame.

$2r$ and L denote the diameter of each wheel and the distance between point P and the wheel, respectively.

Consider the following mobile robot:

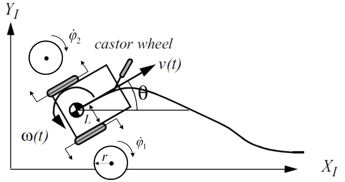

Figure: Mobile robot moving along a trajectory.

$\dot{\varphi}_1$ and $\dot{\varphi}_2$ denote the control inputs of the mobile robot.

$$\dot{\mathbf{x}} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r\dot{\varphi}_1 + r\dot{\varphi}_2 \\ 0 \\ \frac{r\dot{\varphi}_1}{L} - \frac{r\dot{\varphi}_2}{L} \end{bmatrix}, \tag{46}$$

$$\dot{\mathbf{x}} = \begin{bmatrix} r\cos(\theta) \\ r\sin(\theta) \\ r/L \end{bmatrix} \dot{\varphi}_1 + \begin{bmatrix} r\cos(\theta) \\ r\sin(\theta) \\ -r/L \end{bmatrix} \dot{\varphi}_2, \tag{47}$$

$$\dot{\mathbf{x}} = g_1(\mathbf{x})\dot{\varphi}_1 + g_2(\mathbf{x})\dot{\varphi}_2 \tag{48}$$

$$\dot{\mathbf{x}} = g_1(\mathbf{x})\omega_1 + g_2\omega_2, \tag{49}$$

where

$$g_1(\mathbf{x}) \triangleq \begin{bmatrix} r\cos(\theta) \\ r\sin(\theta) \\ r/L \end{bmatrix} \text{ and } g_2(\mathbf{x}) \triangleq \begin{bmatrix} r\cos(\theta) \\ r\sin(\theta) \\ -r/L \end{bmatrix}.$$

First we select the following outputs

$$y_1 = h_1(\mathbf{x}) = x \quad \text{and} \quad y_2 = h_2(\mathbf{x}) = y \quad \text{where} \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ \theta \end{pmatrix}$$

Again let us assume that the relative degree of the system is $(\rho_1, \rho_2) = (1, 1)$.

$$\begin{aligned} \mathbf{A}(\mathbf{x}) &= \begin{bmatrix} \mathcal{L}_{g_1}\mathcal{L}_f^0 h_1(\mathbf{x}) & \mathcal{L}_{g_2}\mathcal{L}_f^0 h_1(\mathbf{x}) \\ \mathcal{L}_{g_1}\mathcal{L}_f^0 h_2(\mathbf{x}) & \mathcal{L}_{g_2}\mathcal{L}_f^0 h_2(\mathbf{x}) \end{bmatrix} = \begin{bmatrix} \mathcal{L}_{g_1} h_1(\mathbf{x}) & \mathcal{L}_{g_2} h_1(\mathbf{x}) \\ \mathcal{L}_{g_1} h_2(\mathbf{x}) & \mathcal{L}_{g_2} h_2(\mathbf{x}) \end{bmatrix} \\ &= \begin{bmatrix} r\cos\theta & r\cos\theta \\ r\sin\theta & r\sin\theta \end{bmatrix}, \end{aligned} \tag{50}$$

We conclude that $\mathbf{A}(\mathbf{x})$ is singular and the relative degree is not $(1, 1)$.

Our system is originally in the following form

$$\dot{\mathbf{x}} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} r\cos\theta\omega_1 \\ r\sin\theta\omega_1 \\ \frac{r}{L}\omega_1 \end{pmatrix} + \begin{pmatrix} r\cos\theta\omega_2 \\ r\sin\theta\omega_2 \\ -\frac{r}{L}\omega_2 \end{pmatrix}$$

Let us add the following variable to make ω_1 appear later in a higher-order derivative of the output

$$\omega_1 = \zeta_1 \tag{51}$$

$$\dot{\zeta}_1 = \tau_1 \tag{52}$$

The new representation of the system is

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\zeta}_1 \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} r\cos\theta\zeta_1 + r\cos\theta\omega_2 \\ r\sin\theta\zeta_1 + r\sin\theta\omega_2 \\ \tau_1 \\ \frac{r}{L}\zeta_1 - \frac{r}{L}\omega_2 \end{pmatrix}$$

$$\dot{\mathbf{x}} = \begin{pmatrix} r\cos\theta\zeta_1 \\ r\sin\theta\zeta_1 \\ 0 \\ \frac{r}{L}\zeta_1 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}\tau_1 + \begin{pmatrix} r\cos\theta \\ r\sin\theta \\ 0 \\ -\frac{r}{L} \end{pmatrix}\omega_2$$

$$= f(\mathbf{x}) + g_1(\mathbf{x})\tau_1 + g_2(\mathbf{x})\omega_1 \quad (53)$$

$$y_1 = h_1(\mathbf{x}) = x \quad \text{and} \quad y_2 = h_2(\mathbf{x}) = y \quad \text{where} \quad \mathbf{x} = \begin{pmatrix} x \\ y \\ \zeta_1 \\ \theta \end{pmatrix}$$

Now we have to calculate $\mathcal{L}_f h_1(\mathbf{x})$ and $\mathcal{L}_f h_2(\mathbf{x})$

$$\mathcal{L}_f h_1(\mathbf{x}) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} r\zeta_1\cos\theta \\ r\zeta_1\sin\theta \\ 0 \\ \frac{r}{L}\zeta_1 \end{bmatrix} = r\zeta_1\cos\theta \quad (54)$$

$$\mathcal{L}_f h_2(\mathbf{x}) = \begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix} \begin{bmatrix} r\zeta_1 \cos\theta \\ r\zeta_1 \sin\theta \\ 0 \\ \frac{r}{L}\zeta_1 \end{bmatrix} = r\zeta_1 \sin\theta \tag{55}$$

Now we calculate the entries of $\mathbf{A}(\mathbf{x})$

$$\mathcal{L}_{g_1}\mathcal{L}_f h_1(\mathbf{x}) = \begin{bmatrix}0 & 0 & r\cos\theta & -r\zeta_1\sin\theta\end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = r\cos\theta \tag{56}$$

$$\mathcal{L}_{g_2}\mathcal{L}_f h_1(\mathbf{x}) = \begin{bmatrix}0 & 0 & r\cos\theta & -r\zeta_1\sin\theta\end{bmatrix} \begin{bmatrix} r\cos\theta \\ r\sin\theta \\ 0 \\ -\frac{r}{L} \end{bmatrix} = \frac{r^2}{L}\zeta_1 \sin\theta \tag{57}$$

$$\mathcal{L}_{g_1}\mathcal{L}_f h_2(\mathbf{x}) = \begin{bmatrix} 0 & 0 & r\sin\theta & r\zeta_1\cos\theta \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} = r\sin\theta \tag{58}$$

$$\mathcal{L}_{g_2}\mathcal{L}_f h_2(\mathbf{x}) = \begin{bmatrix} 0 & 0 & r\sin\theta & r\zeta_1\cos\theta \end{bmatrix} \begin{bmatrix} r\cos\theta \\ r\sin\theta \\ 0 \\ -\frac{r}{L} \end{bmatrix} = -\frac{r^2}{L}\zeta_1\cos\theta \tag{59}$$

Therefore, $\mathbf{A}(\mathbf{x})$ is given by

$$\mathbf{A}(\mathbf{x}) = \begin{bmatrix} r\cos\theta & \frac{r^2}{L}\zeta_1\sin\theta \\ r\sin\theta & -\frac{r^2}{L}\zeta_1\cos\theta \end{bmatrix}, \quad |\mathbf{A}(\mathbf{x})| = -\frac{r^3}{L}\zeta_1 \tag{60}$$

As long as $\zeta_1 \neq 0$, $\mathbf{A}(\mathbf{x})$ is nonsingular, and the relative degree of the system is $(2,2)$.

Vector Relative Degree

Let us now design the control system for the following system:

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = \begin{pmatrix} r\cos\theta & \frac{r^2}{L}\zeta_1\sin\theta \\ r\sin\theta & -\frac{r^2}{L}\zeta_1\cos\theta \end{pmatrix}\begin{pmatrix} \tau_1 \\ \omega_2 \end{pmatrix} + \begin{pmatrix} -\frac{r^2}{L}\zeta_1^2\sin\theta \\ \frac{r^2}{L}\zeta_1^2\cos\theta \end{pmatrix}$$
$$+ \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix}\dot{\omega}_2 + \begin{pmatrix} -\frac{r^2}{L}\zeta_1\sin\theta \\ \frac{r^2}{L}\zeta_1\cos\theta \end{pmatrix}\omega_2 + \begin{pmatrix} \frac{r^2}{L}\sin\theta \\ -\frac{r^2}{L}\cos\theta \end{pmatrix}\omega_2^2$$

Select the following control input:

$$\begin{pmatrix} \tau_1 \\ \omega_2 \end{pmatrix} = \mathbf{A}(\mathbf{x})^{-1}\left(\begin{bmatrix} \frac{r^2}{L}\zeta_1^2\sin\theta \\ -\frac{r^2}{L}\zeta_1^2\cos\theta \end{bmatrix} + \begin{bmatrix} \widehat{\tau}_1 \\ \widehat{\omega}_2 \end{bmatrix}\right) \tag{61}$$

This control yields

$$\begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix}\dot{\omega}_2 + \begin{pmatrix} -\frac{r^2}{L}\zeta_1\sin\theta \\ \frac{r^2}{L}\zeta_1\cos\theta \end{pmatrix}\omega_2 + \begin{pmatrix} \frac{r^2}{L}\sin\theta \\ -\frac{r^2}{L}\cos\theta \end{pmatrix}\omega_2^2 + \begin{bmatrix} \widehat{\tau}_1 \\ \widehat{\omega}_2 \end{bmatrix} \tag{62}$$

Vector Relative Degree

$$\begin{bmatrix} \widehat{\tau}_1 \\ \widehat{\omega}_2 \end{bmatrix} = -\begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix}\dot{\omega}_2 - \begin{pmatrix} -\frac{r^2}{L}\zeta_1 \sin\theta \\ \frac{r^2}{L}\zeta_1\cos\theta \end{pmatrix}\omega_2 - \begin{pmatrix} \frac{r^2}{L}\sin\theta \\ -\frac{r^2}{L}\cos\theta \end{pmatrix}\omega_2^2 - \begin{bmatrix} \ddot{x}_{\rm d} + k_1\dot{e}_x + k_2 e_x \\ \ddot{y}_{\rm d} + k_3\dot{e}_y + k_4 e_y \end{bmatrix} \quad (63)$$

Vector Relative Degree

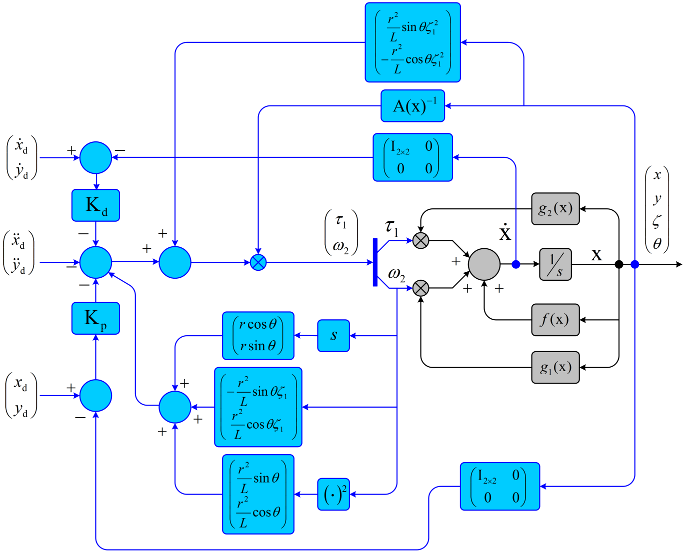

Figure: Control of a tricycle mobile robot via dynamic state feedback.

Thanks

Questions please