Assignment 1 - Deadline 1 week

Problem 1
This exercise is designed to show how small changes in the coefficients of a linear system can affect a fixed point that is a center. Consider the linear system

$$\dot{x}_1 = 0x_1 + x_2 \quad (1)$$
$$\dot{x}_2 = -x_1 + 0x_2 \quad (2)$$

- Find the eigenvalues of the coefficient matrix, classify the fixed point (which is the origin), and determine its stability.

Now consider the linear system

$$\dot{x}_1 = \varepsilon x_1 + x_2 \quad (3)$$
$$\dot{x}_2 = -x_1 + \varepsilon x_2 \quad (4)$$

where ε is arbitrarily small.

- Find the eigenvalues of the coefficient matrix. Show that no matter how small $\mid \varepsilon \mid \neq 0$ is, the center has been changed into a different type of fixed point. What type of fixed point is it?
- Determine the stability of the fixed point for $\varepsilon < 0$ and $\varepsilon > 0$.

Problem 2
This exercise is designed to show how small changes in the coefficients of a linear system can affect the nature of a fixed point when the eigenvalues of the coefficient matrix are equal. Consider the linear system

$$\dot{x}_1 = -x_1 + x_2 \quad (5)$$
$$\dot{x}_2 = 0x_1 - x_2 \quad (6)$$

- Find the eigenvalues of the coefficient matrix, classify the fixed point (which is the origin), and determine its stability.

Now consider the linear system

$$\dot{x}_1 = \varepsilon x_1 + x_2 \quad (7)$$
$$\dot{x}_2 = -\varepsilon x_1 + \varepsilon x_2 \quad (8)$$

where ε is arbitrarily small.

- Find the eigenvalues of the coefficient matrix.
- Classify the fixed point and determine its stability if $\varepsilon > 0$.
- Classify the fixed point and determine its stability if e $\varepsilon < 0$ but $\mid \varepsilon \mid \neq 0$.

Problem 3
For each of the following systems, (i) find the equilibrium points, (ii) classify their type and stability, (iii) then sketch the nullclines, the vector field, and a plausible phase portrait

1. $\dot{x}_1 = x_1 - x_2$, $\dot{x}_2 = 1 - \exp^{x_1}$
2. $\dot{x}_1 = x_1 - x_1^3$, $\dot{x}_2 = -x_2$
3. $\dot{x}_1 = x_1 (x_1 - x_2)$, $\dot{x}_2 = x_2 (2x_1 - x_2)$
4. $\dot{x}_1 = x_2$, $\dot{x}_2 = x_1 (1 + x_2) - 1$
5. $\dot{x}_1 = x_1 (2 - x_1 - x_2)$, $\dot{x}_2 = x_1 - x_2$
6. $\dot{x}_1 = x_1^2 - x_2$, $\dot{x}_2 = x_1 - x_2$

Problem 4
Determine the type and stability of the fixed point at the origin for the nonlinear system

$$\dot{x}_1 = a x_1^3 - x_2 \qquad (9)$$
$$\dot{x}_2 = x_1 + a x_2^3 \qquad (10)$$

for all real values of the parameter a.

Problem 5
For each of the following systems, (i) find the equilibrium points, (ii) classify their type and stability, (iii) then sketch the nullclines, the vector field, and a plausible phase portrait

1. $\dot{x}_1 = x_1 - x_2 + x_1 x_2$, $\dot{x}_2 = 3x_1 - 2x_2 - x_1 x_2$
2. $\dot{x}_1 = x_1 + x_1^2 + x_2^2$, $\dot{x}_2 = x_2 - x_1 x_2$
3. $\dot{x}_1 = -2x_1 - x_2 - x_1(x_1^2 + x_2^2)$, $\dot{x}_2 = x_1 - x_2 + x_2(x_1^2 + x_2^2)$
4. $\dot{x}_1 = x_2 + x_1(1 - x_1^2 - x_2^2)$, $\dot{x}_2 = -x_1 - x_2 + x_2(1 - x_1^2 - x_2^2)$
5. $\dot{x}_1 = 2x_1 + x_2 + x_1 x_2^3$, $\dot{x}_2 = x_1 - 2x_2 - x_1 x_2$
6. $\dot{x}_1 = x_1 + 2x_1^2 - x_2^2$, $\dot{x}_2 = x_1 - 2x_2 + x_1^3$
7. $\dot{x}_1 = x_2$, $\dot{x}_2 = x_2 - x_1 + \mu x_2(1 - x_1^2)$, $\mu > 0$
8. $\dot{x}_1 = 1 + x_1 - \exp^{-x_1}$, $\dot{x}_2 = x_2 - \sin x_1$
9. $\dot{x}_1 = (1 + x_1) \sin x_2$, $\dot{x}_2 = 1 - x_1 - \cos x_2$
10. $\dot{x}_1 = \exp(-x_1 + x_2) - \cos x_1$, $\dot{x}_2 = \sin(x_1 - 3x_2)$