Robotics

Islam S. M. Khalil

German University in Cairo

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Consider the set of linear algebraic equations

$$\mathbf{A}\mathbf{x} = \mathbf{y} \tag{1}$$

where $\mathbf{A}$ and $\mathbf{y}$ are, respectively, $m \times n$ and $m \times 1$ real matrices and $\mathbf{x}$ is an $n \times 1$ vector. The matrices $\mathbf{A}$ and $\mathbf{y}$ are given and $\mathbf{x}$ is the unknown to be solved. Thus the set actually consists of m equations and n unknowns. The number f equations can be larger than, equal, or smaller than the number of unknowns.

Linear Algebraic Equations

Consider the set of linear algebraic equations

$$\mathbf{Ax} = \mathbf{y} \quad (2)$$

Rank of **A**

The range space of **A** is defined as all possible linear combinations of all columns of **A**. The rank of **A** is defined as the dimension of the range space or the number of lineally independent (contributes something new to the space) columns in **A**.

Null Space of **A**

A vector **x** is called a null vector of **A** if **Ax**=0. The null space of **A** consists of all its null vectors.

$$\text{Nullity}(\mathbf{A}) = \text{number of columns of } \mathbf{A} - \text{rank}(\mathbf{A}) \quad (3)$$

Linear Algebraic Equations

Consider the matrix

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 1 & 2 & 3 & 4 \\ 2 & 0 & 2 & 0 \end{bmatrix} = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_4 \end{bmatrix} \quad (4)$$

where $\mathbf{a}_i$ denotes the ith column of $\mathbf{A}$. Clearly $\mathbf{a}_1$ and $\mathbf{a}_2$ are linearly independent. The third column in the sum of the first two columns or $\mathbf{a}_1 + \mathbf{a}_2 - \mathbf{a}_3 = 0$. The last column is twice the second column, or $2\mathbf{a}_2 - \mathbf{a}_4 = 0$. Thus, $\mathbf{A}$ has two linearly independent columns and has rank of 2. The set $(\mathbf{a}_1, \mathbf{a}_2)$ can be used as a basis of the range space of $\mathbf{A}$.

Equation (3) implies that the nullity of $\mathbf{A}$ is 2. It can be verified that the following two vectors form a basis for the null space:

$$\mathbf{n}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathbf{n}_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \\ -1 \end{bmatrix} \quad (5)$$

Thank You

Thank You!
Questions please