Robotics: Tutorial 5

Mechatronics Engineering

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For the shown Delta robot, Derive the equations position, velocity and Acceleration level Kinematics equations (Forward and inverse):

Figure: Delta Robot

The Forward Position Level Kinematic

First Step: Assign the frames: Newtonian Frame and local frames at each joint.

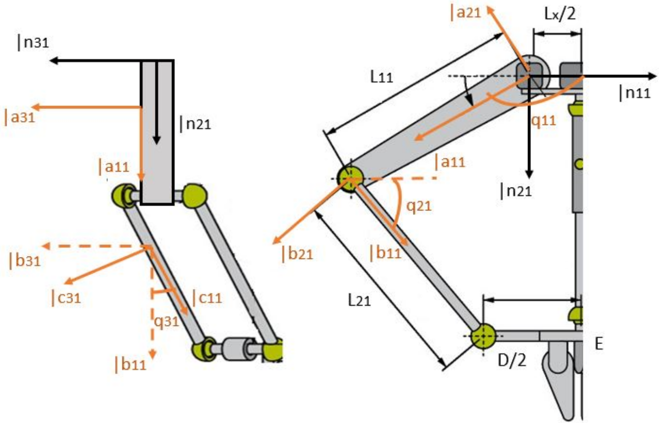

Figure: Side view and front view of Delta robot

Second Step: Write Down the loop equations moving from Origin (O) to the end-effector (E):

$$|r^{OE} = x|n_{11} + y|n_{21} + z|n_{31} \quad (1)$$

$$|r^{OE} = l_{11}|a_{11} + l_{21}|c_{11} + \frac{D}{2}|n_{11} \quad (2)$$

Third Step: Derive the rotation matrices:

	$\vert n_{11}$	$\vert n_{21}$	$\vert n_{31}$
$\vert a_{11}$	$c(q_{11})$	$s(q_{11})$	0
$\vert a_{21}$	$-s(q_{11})$	$c(q_{11})$	0
$\vert a_{31}$	0	0	1

	$\vert n_{11}$	$\vert n_{21}$	$\vert n_{31}$
$\vert b_{11}$	$c(q_{21})$	$s(q_{21})$	0
$\vert b_{21}$	$-s(q_{21})$	$c(q_{21})$	0
$\vert b_{31}$	0	0	1

	$\vert b_{11}$	$\vert b_{21}$	$\vert b_{31}$
$\vert c_{11}$	$cos(q_{31})$	0	$-sin(q_{31})$
$\vert c_{21}$	0	1	0
$\vert c_{31}$	$sin(q_{31})$	0	$cos(q_{31})$

where,

$$|a_{11} = cos(q_{11})|n_{11} + sin(q_{11})|n_{21} \tag{3}$$

For second frame,

$$|b_{11} = cos(q_{21})|n_{11} + sin(q_{21})|n_{21} \tag{4}$$

For third frame,

$$|c_{11} = cos(q_{31})|b_{11} - sin(q_{31})|b_{31} \tag{5}$$

For forth frame,

$$|b_{31} = |n_{31} \tag{6}$$

Substituting equations (3), (4), (5) and (6) in (2), and equating both sides of the equation (1) and (2):

$$x|n_{11} + y|n_{21} + z|n_{31} = l_{11}(cos(q_{11})|n_{11} + sin(q_{11})|n_{21})$$
$$+l_{21}cos(q_{31})(cos(q_{21})|n_{11}+sin(q_{21})|n_{21})-l_{21}sin(q_{31})|n_{31}+\frac{D}{2}|n_{11} \tag{7}$$

Taking common factors for the right side of the equation $(|n_{11}, |n_{21}, |n_{31})$.

The Forward Position level Kinematics in matrix form, so that $X = f(q)$:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} l_{11} cos(q_{11}) + l_{21} cos(q_{31}) cos(q_{21}) + \frac{D}{2} \\ l_{11} sin(q_{11}) + l_{21} cos(q_{31}) sin(q_{21}) \\ -L_{21} sin(q_{31}) \end{bmatrix} \quad (8)$$

Relating Newtonian frames with respect to each other:

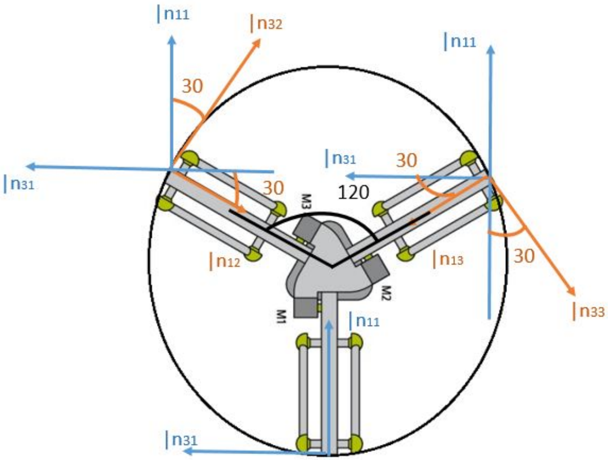

Figure: Top view of delta robot

$$|n_{12} = -sin(30)|n_{11} - cos(30)|n_{31} \tag{9}$$

$$|n_{32} = cos(30)|n_{11} - sin(30)|n_{31} \tag{10}$$

$$|n_{22} = |n_{21} \tag{11}$$

$$|n_{13} = -sin(30)|n_{11} + cos(30)|n_{31} \tag{12}$$

$$|n_{33} = -cos(30)|n_{11} - sin(30)|n_{31} \tag{13}$$

$$|n_{23} = |n_{21} \tag{14}$$

For vector loop equation of second leg:

$$x|n_{12} + y|n_{22} + z|n_{32} = l_{12}(cos(q_{12})|n_{12} + sin(q_{12})|n_{22})$$
$$+l_{22}cos(q_{32})(cos(q_{22})|n_{12}+sin(q_{22})|n_{22})-l_{22}sin(q_{32})|n_{32}+\frac{D}{2}|n_{12} \tag{15}$$

- which is the same as equation (8), but with the variables of the second leg.
- get the equations with respect to the frames n_{11}, n_{21}, n_{31} by substitute equation (9), (10) and (11) in equation (15) and removing frames (n_{12}, n_{22}, n_{32}).

For vector loop equation of third leg:

$$x|n_{13} + y|n_{23} + z|n_{33} = l_{13}(cos(q_{13})|n_{13} + sin(q_{13})|n_{23})$$
$$+l_{23}cos(q_{33})(cos(q_{23})|n_{13}+sin(q_{23})|n_{23})-l_{23}sin(q_{33})|n_{33}+\frac{D}{2}|n_{13} \tag{16}$$

- which is the same as equation (8), but with the variables of the third leg.
- get the equations with respect to the frames n_{11}, n_{21}, n_{31} by substitute equation (12), (13) and (14) in equation (16) and removing frames (n_{13}, n_{23}, n_{33}).

Since q_{21}, q_{31}, q_{22}, q_{23}, q_{32} and q_{33} are passive angles, it needs to be expressed in terms of active angles q_{11}, q_{12} and q_{13} using Holonomic constraint equations:

$$\frac{l_x}{2}|n_{11}+l_{11}|a_{11}+l_{21}|c_{11}+D|n_{11}-l_{22}|c_{12}-l_{12}|a_{12}-\frac{l_x}{2}|n_{12}=0 \quad (17)$$

$$\frac{l_x}{2}|n_{11}+l_{11}|a_{11}+l_{21}|c_{11}+D|n_{11}-l_{23}|c_{13}-l_{12}|a_{13}-l_x|n_{13}=0 \quad (18)$$

$$\frac{l_x}{2}|n_{12}+l_{12}|a_{12}+l_{22}|c_{12}+D|n_{11}-l_{23}|c_{13}-l_{12}|a_{13}-l_x|n_{13}=0 \quad (19)$$

Substituting equations of the local frames and Newtonian frames in equations (17),(18) and (19) for all equations to be with respect to Newtonian frame.

Using Newton Raphson :

$$q_{n+1} = q_n - [\frac{\partial F}{\partial q}]^{-1}|_{q=q_n} F(q_n)$$

$$\begin{bmatrix} q_{21} \\ q_{31} \\ q_{22} \\ q_{32} \\ q_{23} \\ q_{33} \end{bmatrix}_{n+1} = \begin{bmatrix} q_{21} \\ q_{31} \\ q_{22} \\ q_{32} \\ q_{23} \\ q_{33} \end{bmatrix}_{n} - \begin{bmatrix} \frac{\partial F_1}{\partial q_{21}} & .. & .. & .. & .. & \frac{\partial F_1}{\partial q_{33}} \\ .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. \\ \frac{\partial F_6}{\partial q_{21}} & .. & .. & .. & .. & \frac{\partial F_6}{\partial q_{33}} \end{bmatrix} \Bigg|^{-1} \left[f(q_{11}, q_{21}, q_{31}, q_{12}, q_{22}, q_{32}, q_{13}, q_{23}, q_{33})_n \right] \quad (20)$$

where functions from F_1 to F_6 are components of Holonomic constraint equations (17),(18) and (19) in $|n_{11}, |n_{12}$ and $|n_{13}$. Assume initial solution and after several iterations, get value for the passive angles to be substituted in the forward position level kinematics.

The Inverse Position Level Kinematic

Using the Newton Raphson technique:

$$\begin{bmatrix} q_{11} \\ q_{21} \\ q_{31} \\ q_{12} \\ q_{22} \\ q_{32} \\ q_{13} \\ q_{23} \\ q_{33} \end{bmatrix}_{n+1} = \begin{bmatrix} q_{11} \\ q_{21} \\ q_{31} \\ q_{12} \\ q_{22} \\ q_{32} \\ q_{13} \\ q_{23} \\ q_{33} \end{bmatrix}_{n} - \begin{bmatrix} \frac{\partial F_1}{\partial q_{11}} & .. & .. & .. & .. & .. & .. & .. & \frac{\partial F_1}{\partial q_{33}} \\ .. & .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. & .. \\ \frac{\partial F_9}{\partial q_{11}} & .. & .. & .. & .. & .. & .. & .. & \frac{\partial F_9}{\partial q_{33}} \end{bmatrix}^{-1}$$

$$\begin{bmatrix} f(q_{11}, q_{21}, q_{31}, q_{12}, q_{22}, q_{32}, q_{13}, q_{23}, q_{33})_n \end{bmatrix} \quad (21)$$

Assume initial solution and after several iterations, get value for all the angles.

Important Notice

- For f_1, f_2 and f_3 will be from equations x,y and z - equation (8).
- For f_4, f_5 and f_6 from equation (15) with its components in $(|n_1, |n_2, |n_3)$.
- For f_7, f_8 and f_9 from equation (16) with its components in $(|n_1, |n_2, |n_3)$.
- Functions f_1 to f_9 are used in inverse position, forward velocity and forward acceleration

Forward Velocity Level Kinematics

Using Chain rule:

$$\frac{dx}{dt} = \left(\frac{\partial x}{\partial q}\frac{\partial q}{\partial t}\right)$$

- Once for each q, in this case q_{11}, q_{21}, q_{31}, q_{12}, q_{22}, q_{32}, q_{13},q_{23} and q_{33}.
- To get $\dot{x}$, $\dot{y}$ and $\dot{z}$ differentiate f_1, f_2 and f_3 and they will be in terms of passive velocities.
- Therefore differentiate the constraint equations f_4, f_5,f_6,f_7, f_8 and f_9.Get passive velocities in terms of active velocities using matrix form due to linear relation between the velocities.
- Substitute in $\dot{x}$, $\dot{y}$ and $\dot{z}$ to be in terms of active velocities only.
- Finally, place it in a matrix form $\dot{X} = J(q)\dot{q}$.

Inverse Velocity Level Kinematics

Placed in a matrix form so that:

$$\dot{q} = J^{-1}(q)\dot{X}$$

The inverse of Jacobian Matrix can be calculated using MATLAB.

Forward Acceleration Level Kinematics

Using the chain rule:

$$\frac{d\dot{x}}{dt} = \left(\frac{\partial \dot{x}}{\partial q}\frac{\partial q}{\partial t} + \frac{\partial \dot{x}}{\partial \dot{q}}\frac{\partial \dot{q}}{\partial t}\right)$$

- once for each q, in this case q_{11}, q_{21}, q_{31}, q_{12}, q_{22}, q_{32}, q_{13}, q_{23}, q_{33}, $\dot{q}_{11}$, $\dot{q}_{21}$, $\dot{q}_{31}$, $\dot{q}_{12}$, $\dot{q}_{22}$, $\dot{q}_{32}$, $\dot{q}_{13}$, $\dot{q}_{23}$ and $\dot{q}_{33}$).
- To get $\ddot{x}$, $\ddot{y}$ and $\ddot{z}$ differentiate $\dot{x}$, $\dot{y}$ and $\dot{z}$ and they will be in terms of passive accelerations.
- Therefore differentiate $\dot{f}_4, \dot{f}_5, \dot{f}_6, \dot{f}_7, \dot{f}_8$ and $\dot{f}_9$.
- Get passive accelerations in terms of active accelerations by placing it in matrix form due to affine relation between accelerations and substitute in $\ddot{x}$, $\ddot{y}$ and $\ddot{z}$ to be in terms of active accelerations only.
- Finally place it in a matrix form $\ddot{X} = J(q)\ddot{q} + \dot{J}(q)\dot{q}$.

Inverse Acceleration Level Kinematics

The inverse acceleration kinematics solution is:

$$\ddot{q} = J^{-1}(q)[\ddot{X} - \dot{J}(q)\dot{q}]$$

where, $J^{-1}(q)$ Inverse of Jacobian Matrix can be calculated using MATLAB.

Any Questions ?
Thank you