

# Robust Servo Control Using PI-Luenberger Observers with Application to Nonlinear Piezo-Electrically Actuated Systems

Ahmed H. El-Shaer, Mohammad Al Janaideh and Islam S. M. Khalil

**Abstract**—This paper deals with servo control of SISO nonlinear systems driven by piezoelectric actuators operating in highly uncertain conditions. A proportional-integral (PI) Luenberger observer-based controller is systematically designed to achieve accurate tracking performance, enforce desired closed loop dynamics, as well as robust disturbance rejection in the presence of nonlinearities and model uncertainties. The controller presented offers many advantages including simplicity, ease-of-tuning and implementation in wide variety of applications involving piezoelectric-based actuators. Closed loop experimental results show tracking performance robustness is indeed realized in the presence of the plant nonlinearities and the actuator hysteresis.

## I. INTRODUCTION

This paper is concerned with robust servo control of SISO nonlinear systems driven by piezoelectric actuators. The system dynamics include both actuator and plant nonlinearities making the analysis and control design more general. Piezoelectric actuators are becoming increasingly more popular for different applications including positioning and manipulating objects at nano-scale, atomic force microscopy, fuel injection and many others [1],[2],[3],[4]. Refer to the survey [5] for a more comprehensive list of recent applications. The key component in most of these applications is the piezoelectric actuator which functions as an electro-mechanical transformer of electric voltage into mechanical forces/torques [1]. However, the voltage-to-displacement behavior is inherently hysteretic in nature which degrades the actuator accuracy and could lead to closed loop limit-cycle oscillations, see [6] and the references therein.

Recently, numerous control methods have been developed for hysteresis compensation. They can generally be described as (i) inverse, and/or (ii) non-inverse-based hysteresis compensation. The former approach is comprised of a cascade of a hysteresis inverse model and application-dependent outer controller [6]. On the other hand, non-inversion-based hysteresis compensation employs an “approximate” hysteresis model in the control design [7]. In either type, there are many feedback control strategies including PID-type controllers,  $H_{\infty}$ /loop-shaping, sliding mode and nonlinear adaptive controllers to name a few. Refer to [5],[6],[7],[8],[9] and the references therein.

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In most of these works, the actuator is treated as a stand-alone dynamical system made of linear dynamics preceded by a nonlinear hysteresis block. This, however, neglects the coupling between the actuator and the plant which inevitably happens in the aforementioned applications. Further, hysteresis characteristics and mechanical stiffness of piezoelectric actuators vary significantly under mechanical loading and temperature variations imposed by the environment they are intended to operate in [10][11]. Thus, achieving realistic robust hysteresis compensation and reference tracking over wide range of operating conditions, it is necessary to design the closed loop system to be robust against effects introduced by plant dynamics.

This paper presents a unified approach to robust servo control of nonlinear systems involving piezoelectric actuators. The closed loop is composed of a proportional-integral (PI) Luenberger observer-based controller to achieve desired tracking performance and disturbance rejection in presence of plant parametric uncertainty and nonlinearities as well as the actuator hysteresis. PI Luenberger observers have good robustness properties which make them attractive for many classes of nonlinear and uncertain systems [12],[13]. Motivated by recent work on robust disturbance observers (DOBs) [6],[14], connection between classic DOBs and extended state observers is established [15]. Using the latter, a Luenberger observer is designed in this paper based on the internal model principle [16][17] for output feedback servo control. The main contribution of this paper is twofold: (1) theoretically, a unified, largely self-contained treatment of robust extended-state observers is presented for servo control. Necessary and sufficient conditions are given for the existence of such observers for SISO systems, based on mild conditions on the nominal plant dynamics. Also, the robustness analysis presented is sufficiently general to be applicable to a wide class of uncertain feedback systems, (2) practically, the resulting controller is of the classic observer-based architecture which can be simply computed as static gains, making it easy-to-tune and readily implementable in many industrial/research applications. Further, the experimental results given indicate that the feedback system did achieve robust tracking performance and *non-inverse-based* hysteresis compensation.

## II. PZT ACTUATOR MODELING AND EXPERIMENTAL VALIDATION

### A. Lumped Electromechanical Modeling

The PZT actuator studied in this paper is depicted in Fig. 1. In this figure, a power amplifier is used to produce the

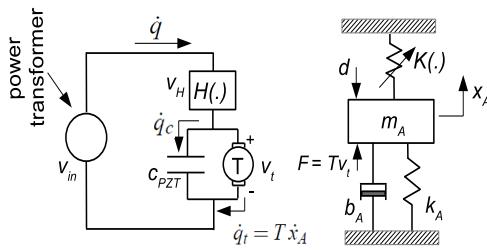


Fig. 1. Lumped electromechanical dynamics of the PZT actuator with nonlinear spring  $K(.)$ .

input voltage  $v_{in}$  across the PZT<sup>1</sup>. The equations of motion presented here follow the dc-motor equivalent model given in Goldfarb et. al. [1] for a lumped-parameter electromechanical system for the PZT actuator, see also [18] and the references therein. The main focus of the subsequent analysis is robust servo control design. The PZT dynamical equations are

#### Mechanical sub-system:

$$m_A \ddot{x}_A + b_A \dot{x}_A + k_A x_A = F - K(x_A) - d, \quad (1a)$$

#### Electrical sub-system:

$$\begin{aligned} F &= T v_t = T(v_{in} - v_H), \\ \dot{q} &= T \dot{x}_A + \dot{q}_c, \\ v_{in} &= V_g \frac{D(e_v)}{1-D(e_v)}, \\ e_v &= u - v_{in}, \end{aligned} \quad (1b)$$

In (1a),  $x_A$  is the measured PZT stack endpoint displacement,  $m_A$ ,  $b_A$  and  $k_A$  are the lumped mass, damping and stiffness, respectively,  $F$  is the actuation force,  $K(x_A)$  represents a nonlinear spring force mechanism and  $d$  is an exogenous disturbance. In (1b),  $T$  is the electromechanical transformer ratio constant,  $v_{in}$  is the (averaged) input voltage,  $v_H$  is the voltage drop across the hysteresis nonlinearity  $H(q)$  and  $q$  is the PZT total charge. The input voltage  $v_{in}$  is related to the source voltage  $V_g$  by the scaling factor  $\frac{D(e_v)}{1-D(e_v)}$  [19], where  $D(e_v)$  is the amplifier duty cycle as a function of the voltage tracking error  $e_v$ . In the sequel, the reference voltage  $u$  is considered the control input to the PZT actuator plant.

It is noted that (1) includes two main nonlinearities: the PZT hysteresis  $H(.)$  and the nonlinear spring force  $K(.)$ . The combined effect of both nonlinearities can be characterized experimentally by plotting  $x_A$  vs.  $v_{in}$  for different desired inputs  $u(t)$ . The data plotted in Fig. 2 reveals dead-zone like behaviour and significant hysteresis in the PZT motion. The presence of these nonlinear effects as well as exogenous disturbances  $d$  (1a) will be dealt with in the controller presented in subsequent sections.

For the purposes of servo-control design, the physical parameters in (1) can be adjusted to fit experimental data. However, with limited available measurements, namely  $x_A$ , and the presence of the nonlinearities:  $H(.)$  and  $K(.)$  whose

<sup>1</sup>Details of the voltage feedback loop are not needed in this work, hence will not be discussed further. Extensive analysis of these circuits is found in [19].

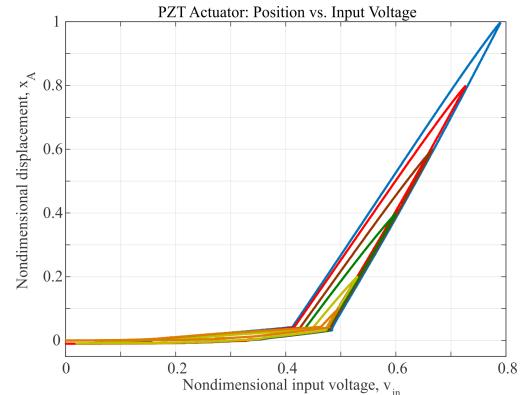


Fig. 2. Combined nonlinear effect of  $K(x_A)$  and  $H(q)$

structures are assumed unknown *a priori*, individual parameter estimation might not be simple. Instead, input-output transfer function identification is employed to capture the underlying dynamics from  $u$  to  $x_A$ .

#### B. Transfer Function Identification and Validation

Using experimental data, it is desired to approximate the PZT actuator dynamics from the input voltage  $u$  to the output position  $x_A$  as an LTI system of given order. This can be efficiently done using the system identification toolbox in Matlab [22], for excitation inputs  $u(t) = A \sin(\omega t) \cos(2\pi(f_0 + \beta t)t)$ , with excitation frequency  $f = f_0 + \beta t$ . In the absence of exogenous disturbance  $d$ , the following normalized responses are obtained, corresponding to  $A = 0.25$  and  $0.5$ , respectively.

The transfer function estimates in Fig. 3<sup>2</sup> are

$$P_1(s) = \frac{1}{0.006s + 1}, \quad P_2(s) = \frac{1}{0.04s + 1} \quad (2)$$

Clearly, the bandwidth of the obtained transfer functions varies with the input amplitude. Further, as Fig. 2 indicates, small  $x_A$  values are largely affected by dead-zone like spring forces. This nonlinear behavior can not be predicted by the transfer functions (2) as shown in Fig. 4.

#### C. Actuator Uncertain Dynamics

Let  $x_A$  be denoted by  $x_p$ , the discussion above suggests that the PZT actuator plant (1) can be approximated as

$$\dot{x}_p = (A_p + \Delta A_p) x_p + B_p(u + d + \Delta(x_p, u)), \quad (3a)$$

$$y_p = C_p x_p, \quad (3b)$$

where  $x_p(t) \in \mathbb{R}$  is the PZT displacement,  $d(t) \in \mathbb{R}$  is the exogenous disturbance, and  $\Delta(x_p, u)(t) \in \mathbb{R}$  is the combined effect of  $H(.)$  and  $K(.)$ . The uncertain system (3a) is nominally linear with perturbations from the parametric (time-varying) uncertainty  $\Delta A_p(t)$  due to bandwidth variation, and the nonlinear term  $\Delta(\cdot, \cdot)$ . This model is used for robust performance analysis in subsequent sections.

<sup>2</sup>The command *pem* is used from the system identification toolbox.

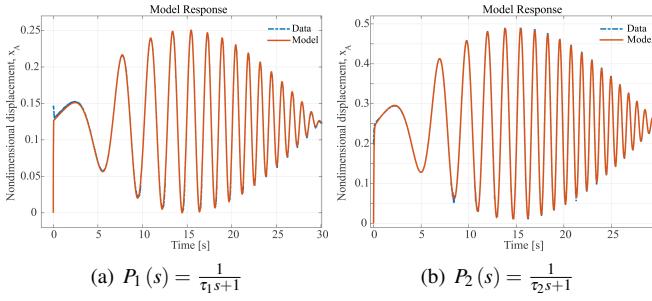


Fig. 3. Model response vs. experimental data

### III. SERVO CONTROL SYSTEM

The rest of this paper is concerned with (3), where the functional form of  $\Delta(\cdot, \cdot)$  is assumed to be *unknown or not exactly known*. Henceforth, for sake of generality, it is assumed that  $A_p \in \mathbb{R}^{n_p \times n_p}$ . Further, it is assumed that  $\Delta(\cdot, \cdot)$  is piecewise continuous and satisfies

- (A1) The map  $t \mapsto \Delta(x_p(t), u(t)) \in L_\infty(\Omega)$ ,  $\Omega \subseteq \mathbb{R}$ , such that

$$\|\Delta\|_{L_\infty} := \sup_{\{[x_p(t) \ u(t)]^T \in \mathbb{R}^{n_p+1}; t \geq 0\}} |\Delta(x_p(t), u(t))| < \infty. \quad (4)$$

The assumption (A1) requires minimal knowledge about the nonlinearity  $\Delta(\cdot, \cdot)$ . Moreover, the bound (4) can be satisfied by many nonlinearities arising in wide variety of dynamical systems including flexible-joint robots, classes of chaotic systems as well as a number of play-stop hysteresis operators, see [15][23] and the references therein.

#### A. Extended-State Observer-Based Controller

Let the nominal plant dynamics be given by the SISO transfer function

$$P_n(s) = B_p(sI - A_p)^{-1} C_p, \quad (5)$$

where it is assumed that

- (A2) The pair  $(A_p, B_p)$  is stabilizable.

Define the augmented state vector  $\hat{x} := \begin{bmatrix} x_p \\ x_w \end{bmatrix}$ , where  $x_w(t) \in \mathbb{R}^{n_w}$  is the state of the exo-system defined below. The control objective is to achieve robust asymptotic reference tracking

$$\lim_{t \rightarrow \infty} |r(t) - y_p(t)| = 0, \quad (6)$$

where  $r$  is a bounded reference input. The control input  $u$  is produced by the observer-based compensator [20]

$$\dot{\hat{x}} = \tilde{A}\hat{x} + \tilde{B}u + L(y_p - \tilde{C}\hat{x}),$$

$$\tilde{A} = \begin{bmatrix} A_p & B_p C_w \\ 0 & A_w \end{bmatrix}; \tilde{B} = \begin{bmatrix} B_p \\ 0 \end{bmatrix}; \tilde{C} = \begin{bmatrix} C_p & 0 \end{bmatrix}, \quad (7a)$$

$$u = [-K_x \ -C_w]\hat{x} + K_r r, \quad (7b)$$

where  $L \in \mathbb{R}^{n_p+n_w}$  is the observer gain,  $K_x \in \mathbb{R}^{1 \times n_p}$  is chosen to set the eigenvalues of  $A_p - B_p K_x$  at desired locations in the open left-half plane and  $K_r$  is a scaling gain to ensure steady state tracking. Without any loss of generality,  $K_r = 1$  for normalized plant dynamics studied in this paper. The

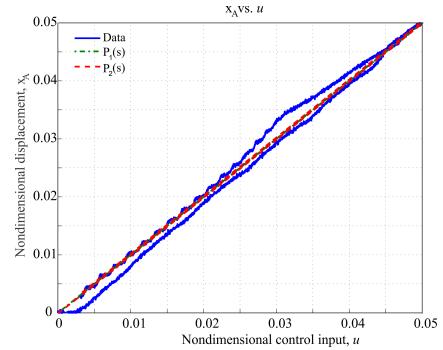


Fig. 4. Hysteresis for low actuator displacements

augmented system (7a) is comprised of (5) and an exo-system to generate the *fictitious* disturbance/unknown input  $w(t) \in \mathbb{R}$  as follows

$$\dot{x}_w = A_w x_w, \quad w = C_w x_w; \quad x_w(t_0) = x_{w0} \in \mathbb{R}^{n_w}. \quad (8)$$

In particular, the control law (7b) can be expressed as  $u = -K_x \hat{x}_p - \hat{w} + K_r r$  which incorporates the estimate  $\hat{w}$  to achieve disturbance rejection. The following assumptions are made

- (A3) The pair  $(A_p, C_p)$  is detectable,
- (A4) The pair  $(A_w, C_w)$  is detectable,
- (A5) The eigenvalues of  $A_w$  don't coincide with the zeros of  $P_h(s)$ .

Assumption (A5) states that the disturbance state  $x_w$  is observable from the output  $y_p$ . The existence of the observer gain  $L$  is addressed in the following theorem.

**Theorem 1** Assumptions (A3)–(A5) are necessary and sufficient conditions for

$$\text{rank} \begin{bmatrix} \lambda I - \tilde{A} \\ \tilde{C} \end{bmatrix} = n_p + n_w \quad (9)$$

for all eigenvalues  $\lambda \in \mathbb{C}$  of  $\tilde{A}$ . Then, there exists  $L \in \mathbb{R}^{n_p+n_w}$  such that

$$\dot{\hat{x}} = (\tilde{A} - L\tilde{C})\hat{x} + [\tilde{B} \quad L] \begin{bmatrix} u \\ y_p \end{bmatrix} \quad (10)$$

is an asymptotically stable Luenberger observer for the augmented system (7a).

**Proof:** It is noted that (10) follows directly from (9) using the PBH observability theorem, [20] p. 256. The rest of the proof can be found in the literature, see [15][14][12] and the references therein.

Define the equivalent lumped disturbance  $d_{eq} := d + \Delta(x_p, u)$ . Assume for appropriately chosen  $A_w$  that  $d_{eq}$  can be expressed as

$$d_{eq} = w + \delta = C_w x_w + \delta, \quad (11)$$

where  $\delta(t) \in \mathbb{R}$  is a norm-bounded approximation error; that is,  $|\delta|_\infty := \sup_{t \geq 0} |\delta(t)| < \infty$ . Define  $\tilde{x} := \begin{bmatrix} \tilde{x}_p \\ \tilde{x}_w \end{bmatrix} := \begin{bmatrix} x_p - \hat{x}_p \\ x_w - \hat{x}_w \end{bmatrix} \in \mathbb{R}^{n_p+n_w}$ , the nominal closed loop comprised of (3a) (with

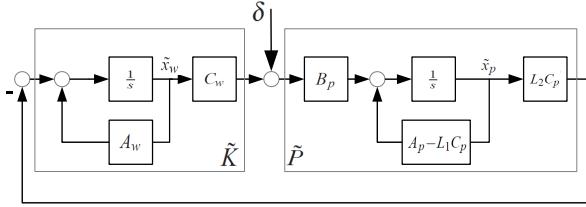


Fig. 5. Exo-System as feedback interconnection of  $\tilde{P}$  and  $\tilde{K}$

$\Delta A_p = 0$ ), (7b), (10), together with (11) is written as

$$\dot{x}_{cl} := \begin{bmatrix} \dot{x}_p \\ \tilde{x} \end{bmatrix} = \underbrace{\begin{bmatrix} A_p - B_p K_x & B_p \bar{K} \\ 0 & \tilde{A} - L \tilde{C} \end{bmatrix}}_{A_{cl}} \begin{bmatrix} x_p \\ \tilde{x} \end{bmatrix} + \underbrace{\begin{bmatrix} K_r B_p \\ 0 \end{bmatrix}}_{B_{cl}} r + \begin{bmatrix} B_p \\ \tilde{B} \end{bmatrix} \delta, \quad (12)$$

where  $\bar{K} := [K_x \ C_w]$ . Thus,  $K_x$  and  $L$  can be separately designed for internal (exponential) stability of the nominal closed loop.

### B. Adding Integral Action

It is noted from Eq. (12) that  $\tilde{x}$  isn't controllable from the reference  $r$ , and is influenced only by  $\delta$ . The exo-system (7a), as subsystem in (12) can be represented by the negative feedback interconnection of

$$\tilde{P}(s) := L_2 C_p (sI - (A_p - L_1 C_p))^{-1} B_p, \quad (13a)$$

$$\tilde{K}(s) := C_w (sI - A_w)^{-1}, \quad (13b)$$

$$L = [L_1^T \ L_2^T]^T, \quad (13c)$$

as depicted in Fig. 5. Consequently, by virtue of the internal model principle [16], the effect of  $\delta$  on the closed loop tracking performance is reduced by including its dynamic characteristics into  $A_w$ . However, in absence of *a priori* knowledge of the spectral characteristics of the disturbance  $\delta$  (e.g. norm-bounded nonlinearities), it can be approximated as “piece-wise” constant input, resulting in at least one integrator in the exo-system dynamics to ensure robust disturbance rejection. This implies, according to theorem 1, that  $P_n(s)$  can not have any zeros at the origin.

*Remarks:*

- The synthesis of an optimal Luenberger observer (10) can be formulated as a weighted-sensitivity  $H_\infty$  optimization of a fixed-structure controller. This gives rise to a *rank-constrained non-convex* optimization, which can be iteratively solved using techniques presented in [15]. In this paper, robust eigenvalue placement is used to compute the controller gains  $K_x$  and  $L$ .
- The inclusion of integral action in the exo-system is key in the DOB-based *performance recovery* framework presented in [6][15]. This enables DOB-based controllers to reject signals not having well defined spectral content. This also includes the class of exogenous disturbances represented by polynomial in-time models, i.e.,  $w(t) = \sum_{n=0}^N w_n t^n$ , which can be represented by higher order disturbance models e.g.,  $w(s) = \frac{1}{s^k}$ ,  $k > 1$ .

### IV. CLOSED LOOP ROBUSTNESS

The uncertainty  $\Delta A_p(t)$  in Eq. (3) is assumed to have norm-bounded, time-varying representation

$$\Delta A_p(t) = E_1 F(t) E_2, \quad (14a)$$

$$\|F(t)\| \leq 1, \quad (14b)$$

where  $E_1$ ,  $F(t)$  and  $E_2$  are matrices of appropriate dimensions. Bounded stability of the closed loop system (12) in the presence of (14) is now established.

**Theorem 2** Suppose that  $K_x$  and  $L$  are chosen such that the eigenvalues of (12) have strictly negative real parts located sufficiently far in the open left-half plane. Then the closed loop system is uniformly bounded input-to-state stable in the presence of (14).

**Proof:** The solution of the closed loop system (12) with (14) is

$$x_{cl}(t) = e^{A_{cl}t} x_{cl}(0) + \int_0^t e^{A_{cl}(t-\tau)} (\psi_1(\tau) + \psi_2(\tau)) d\tau, \quad (15a)$$

$$\psi_1(t) := \begin{bmatrix} K_r B_p \\ 0 \end{bmatrix} r(t) + \begin{bmatrix} B_p \\ \tilde{B} \end{bmatrix} \delta(t), \quad (15b)$$

$$\psi_2(t) := -(E_1 F(t) E_2) x_{cl}(t). \quad (15c)$$

Given the linearity of Eq. (15a), it can be decomposed into  $x_{cl}(t) = x_1(\psi_1)(t) + x_2(x_{cl}(0); \psi_2)(t)$ . The first term satisfies

$$|x_1(t)| \leq \frac{2k}{\alpha} \sup_{t \geq 0} |\psi_1(t)|, \quad (16)$$

where  $|.|$  is the standard Euclidean norm,  $\|.\| := \sqrt{\lambda_{max}\{(.)^T(.)\}}$  is the maximum singular value operator norm,  $k > 0$  and  $\alpha > 0$  are such that  $\|e^{A_{cl}t}\| \leq k e^{-\alpha t}$ , [21] p. 134, and  $|\psi_1|_\infty \leq \|B_p\| (|K_r| |r(t)|_\infty + \sqrt{2} |\delta|_\infty)$ . On the other hand, from (14),  $x_2$  satisfies  $|x_2(t)| \leq k e^{-\alpha t} |x_2(0)| + \int_0^t k \|E_1\| \|E_2\| e^{-\alpha(t-\tau)} |x_2(\tau)| d\tau$ . This implies that

$$|x_2(t)| e^{\alpha t} \leq k |x_2(0)| + \int_0^t k \|E_1\| \|E_2\| e^{\alpha \tau} |x_2(\tau)| d\tau. \quad (17)$$

Applying the Gronwall-Bellman inequality to (17), [20] p. 29, [21] p. 651, it follows that

$$|x_2(t)| \leq k |x_2(0)| e^{-(\alpha - k \|E_1\| \|E_2\|)t}. \quad (18)$$

Hence,  $|x_2(t)|$  is bounded if  $\alpha = k \|E_1\| \|E_2\|$ , and tends to zero if  $\alpha > k \|E_1\| \|E_2\|$ . Both (16) and (18) guarantee the closed loop (12) to be uniformly bounded in the presence of the parametric uncertainty (14).

The condition (18) can be readily satisfied via eigenvalue placement control synthesis. Alternatively, if there exists  $\lambda^* > 0$  such that  $\|e^{-\lambda^* t} e^{(A_{cl}+E_1 F(t) E_2)t}\| < \infty \quad \forall t \geq 0$ , or tends to zero as  $t \rightarrow \infty$ , then the closed loop system (12) is uniformly bounded. Using semi-definite programming techniques, this can be expressed as sufficient robust-stability feasibility conditions for given  $K_x$  and  $L$  [6][15][13]. However, the results in the next section indicate that robust performance

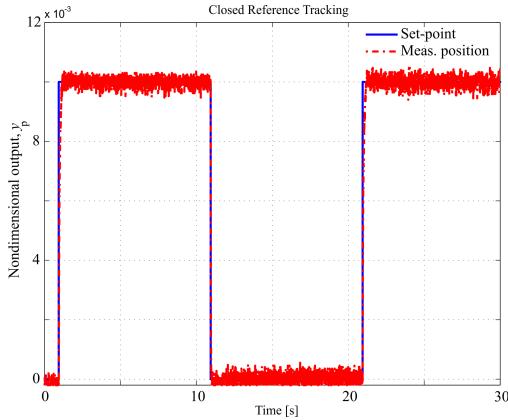


Fig. 6. Step-input tracking: amplitude of 0.01

can be achieved for the closed loop eigenvalues designed to have sufficiently negative real parts.

## V. EXPERIMENTAL RESULTS

The nominal plant and exo-system dynamics are chosen, respectively, as  $P_h(s) := C_p(sI - A_p)^{-1}B_p = \frac{40}{s+40} = \frac{1}{0.025s+1} =: \frac{1}{\tau s+1}$ , and  $A_w = 0$ ,  $C_w = 1$ , i.e.  $w(s) = \frac{1}{s}$ . The gain  $K_x = 10$  is chosen to set the eigenvalue of  $A_p - B_p K_x$  at  $-50$ , and the observer gain  $L = [42.26458 \quad 394.7842]^T$  is chosen to place both eigenvalues of  $\tilde{A} - L\tilde{C}$  at  $-(2\pi)20$ .

It is clear that assumptions (A2)–(A5) in theorem 1 are satisfied. Also, the controller gains are internally stabilizing for practically all  $\tau > 0$  values encountered in this work. The controller is discretized at  $2\text{KHz}$  and implemented using the Simulink real-time toolbox/xPC Target environment within Matlab R2013a.

Closed loop tracking responses are obtained for normalized set-point inputs  $r(t) \in [0 \quad 1]$  as shown in the figures below. In all cases the output  $y_p$  is the normalized position of the PZT endpoint measured by a strain gauge. It is clear that consistent tracking performance is successfully achieved for different reference input amplitudes and waveforms. Also, as shown in Fig. 7, the PZT hysteresis effect is linearized in the reference-to-output closed loop response.

## VI. CONCLUSIONS

This paper is concerned with robust servo control of a class of uncertain SISO nonlinear systems involving actuator hysteresis and plant nonlinearities. An extended-state observer-based controller is presented to achieve robust tracking performance. Necessary and sufficient conditions for the existence of the observer are given and robust stability analysis is presented. Experimental results of a PZT actuated system show that closed loop robust tracking performance is indeed achieved.

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