Assignment 1 - Deadline 1 week

Problem 1

The planar manipulator (Figure 1) has 3 links A and B with length l_1 and l_2, respectively. Links A and B are actuated and described using the generalized coordinates q_1 and q_2, respectively. Derive the following:

- The kinematic equations of the manipulator in the position, velocity, and acceleration levels.
- Derive the forward kinematics (relation between the task space and joint space, i.e., $\mathbf{f}(\mathbf{q})$.
- Derive the Jacobian matrix ($\mathbf{J}(\mathbf{q})$) and calculate its inverse.
- Derive the relation between the acceleration in the task and joint spaces (forward and inverse).

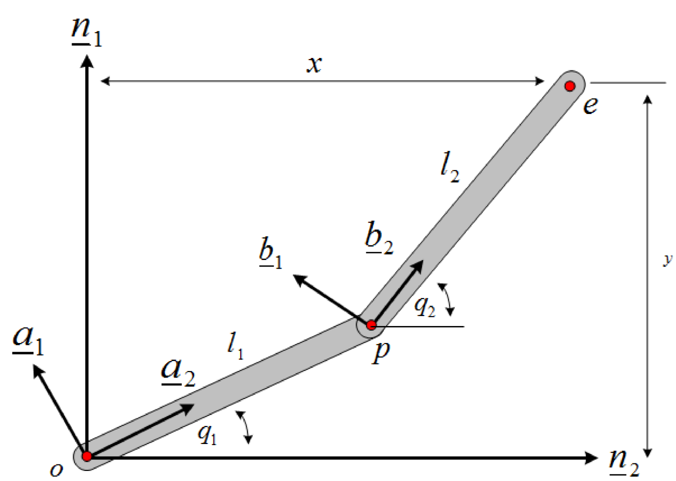

Figure 1: A planar RR robot

Problem 2

Consider the planar RP robot shown in Figure 2, where l is the length of the first link and the generalized coordinates to be used are indicated. Let $\mathbf{p} = (p_x \quad p_y)^{\mathrm{T}}$ be the position of the end-effector P:

- Find the forward kinematic problem of this robot by assigning frames to each body and defining the rotation matrices between these frames;
- Solve the inverse kinematic problem for this robot, providing the number and type of the solutions for varying positions of P;
- Draw the robot primary workspace (with dimensions) in the case when the joint variables are bounded as: $q_1 \in [-\frac{\pi}{2}, +\frac{\pi}{2}]$, $q_2 \in [-L, +L]$. Discuss the presence of singularities on the boundaries of the workspace.

Problem 3

Consider the planar RPR robot shown in Figure 3, and the definition of joint variables $\mathbf{q} = (q_1 \quad q_2 \quad q_3)^{\mathrm{T}}$ given therein. The three-dimensional task vector is $\mathbf{r} = \left(\mathbf{p}^{\mathrm{T}} \quad \alpha\right)^{\mathrm{T}}$, where $\mathbf{p} = (p_x \quad p_y)^{\mathrm{T}}$ is the position of the end-effector and α is the orientation angle of the last link w.r.t. the x_0 axis. Assume that $q_2 \geq 0$ holds for the prismatic joint variable.

- Find the forward kinematic problem of this robot by assigning frames to each body and defining the rotation matrices between these frames;

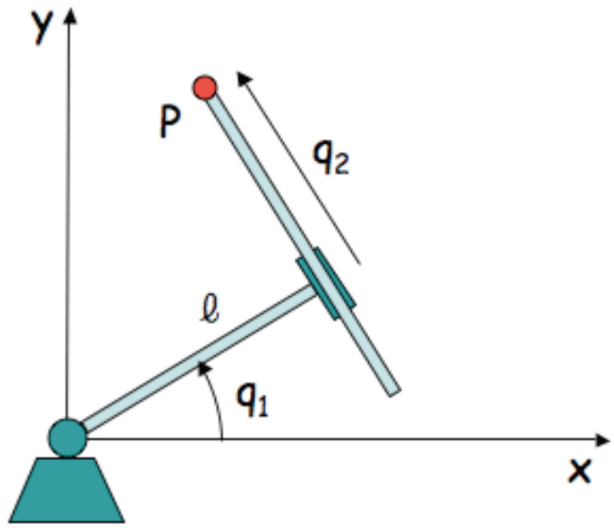

Figure 2: A planar RP robot

- Solve the inverse kinematics problem for a given $\mathbf{r}_\mathrm{d}$, providing the expression of all feasible solutions in closed form.
- Compute the solutions q associated to $\mathbf{r}_\mathrm{d} = (-2 \quad -2 \quad \pi/2)^\mathrm{T}$ (i.e., such that $\mathbf{f}(\mathbf{q}) = \mathbf{r}_\mathrm{d}$) using the data L_1=1 [m] and L_3 = 0.7 [m], and sketch the robot configurations.
- Draw the primary workspace in the plane of robot motion for generic values of L_1 and L_3, assuming that the prismatic joint range is bounded as $| q_2 | \leq D$ (with $D > 0$).

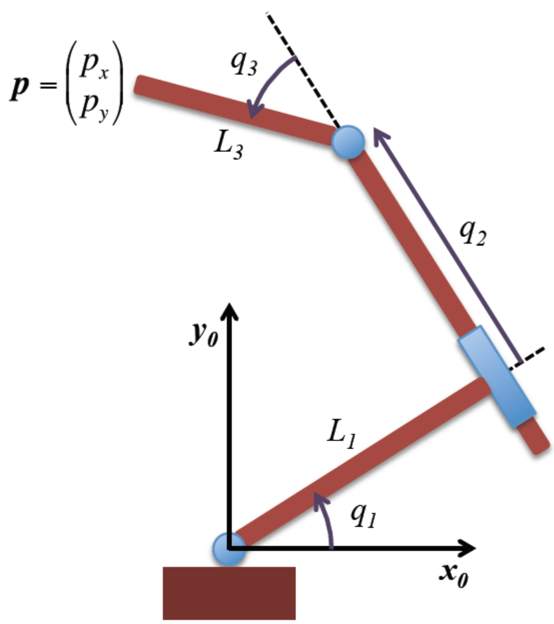

Figure 3: A planar RPR robot with the definition of joint variables