

Advanced Mechatronics Engineering

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November 13, 2016

- Introduction
- Disturbance estimation
- Disturbance rejection
- Disturbance observer-based control systems



Figure: Disturbance in terms of parameter deviation.



Figure: Disturbance in terms of external force input.

Disturbance Estimation

Since plant perturbation and disturbance affect control systems, the disturbance observer shown in Fig. 1 has been utilized to realize a nominal system. The plant has a transfer function $P(s)$ and is influenced by a disturbance d such as friction force, gravity force, and so on. The output y is position, speed, or force depending on an available sensor, which is affected by a sensor noise ξ . The disturbance observer estimates the disturbance by using the inverse nominal plant $P_n^{-1}(s)$ as follows:

$$d_o = u = P_n^{-1}(s)(y - \xi) \quad (1)$$

$$= (P^{-1}(s) - P_n^{-1}(s))y + d + P_n^{-1}(s)\xi \quad (2)$$

$$= \Delta P(s)y + d + P_n^{-1}(s)\xi. \quad (3)$$

where $\Delta P(s)$ in the first term is the perturbation of the real plant from the nominal plant. Therefore, the disturbance observer estimates not only the disturbance but also the perturbation of the real plant from the nominal plant.

Disturbance Estimation

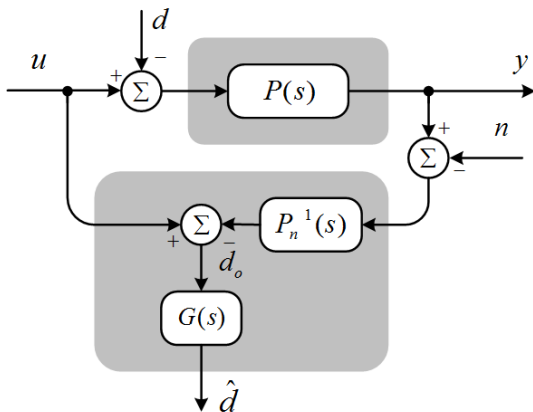


Figure: Disturbance force observer. $P(s)$ and $P_n(s)$ represent the actual and nominal models of the plant, respectively. u and y represent the input and output, respectively. d denotes the disturbance force input, whereas \hat{d} is its estimate.

Disturbance Estimation

Since the inverse nominal plant $P_n^{-1}(s)$ includes some derivatives, the disturbance observer estimates the disturbance d_o through a low pass filter $G(s)$ as follows:

$$\hat{d}_o = G(s)d_o. \quad (4)$$

Model of disturbance is given by

$$\frac{d^k}{dt^k} d_o(t) = 0. \quad (5)$$

The disturbance is approximated by a step (zeroth order) function, a ramp (first order) function, and a parabolic (second order) function when $k = 1$, $k = 2$, and $k = 3$, respectively. When the disturbance observer is designed by the Gopinaths method, $G(s)$ is given as shown in Table I, where $P_n(s)$ includes two integrators (the position-based or force-based disturbance observer).

Table: DESIGNED $G(s)$ IN POSITION-BASED AND FORCE-BASED DISTURBANCE OBSERVERS.

Disturbance observer	$G(s)$
Step ($k = 1$)	$\frac{g_2}{s^2 + g_1 s + g_2}$
Ramp ($k = 2$)	$\frac{g_2 s + g_3}{s^3 + g_1 s^2 + g_2 s + g_3}$
Parabolic ($k = 3$)	$\frac{g_2 s^2 + g_3 s + g_4}{s^4 + g_1 s^3 + g_2 s^2 + g_3 s + g_4}$

Here, the disturbance observer is called by the input and the disturbance model. For example, the disturbance observer which has position input and is designed by the step disturbance model is called the position-based step disturbance observer.

Disturbance Estimation and Compensation

If the estimated disturbance \hat{d}_o is fed back as shown in Fig. Fig. 2, the output y is given by

$$y = \frac{u^{ref} - (1 - G(s))d + G(s)P_n^{-1}(s)\xi}{G(s)P_n^{-1}(s) + (1 - G(s))P_n^{-1}(s)} \quad (6)$$

$$= \frac{P_n(s)(u^{ref} - (1 - G(s))d) + G(s)\xi}{1 + (1 - G(s))\Delta P(s)P_n(s)} \quad (7)$$

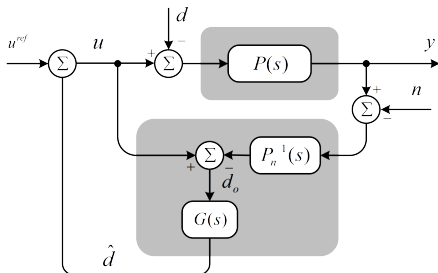


Figure: Disturbance compensation.

This equation shows the following features:

- $G(s)$ represents a sensitivity to the sensor noise.
- $1 - G(s)$ represents a sensitivity to the disturbance and the plant perturbation.

If $(1 - G(s))d = 0$ and $(1 - G(s))\Delta P(s)P_n(s) = 0$, the nominal system is realized as $y = P_n(s)u^{ref}$. $G(s)$ has to be determined according to the characteristic of the disturbance d and the plant perturbation $\Delta P(s)$ so that $(1 - G(s))d$ and $(1 - G(s))\Delta P(s)P_n(s)$ fulfill a specification. Since the plant perturbation $\Delta P(s)$ affects stability of the control system, it is desirable to make $\Delta P(s)$ small, which is realized by determining the nominal plant $P_n(s)$ close to the plant $P(s)$.

Disturbance Estimation and Compensation

On the other hand, since $G(s)$ is the sensitivity to the sensor noise, $G(s)$ has to be determined so that influence of the sensor noise does not become excessively large. Therefore, the bandwidth of the low-pass filter $G(s)$ is limited by the bandwidth of the sensor noise ξ .

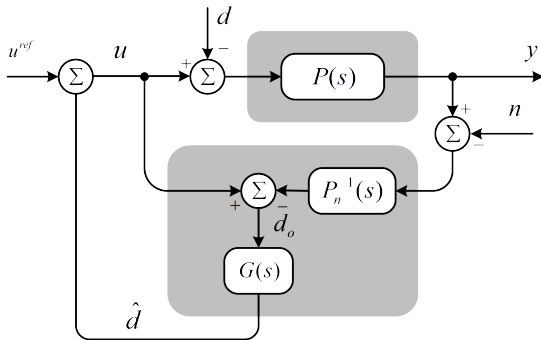


Figure: Disturbance compensation.

Implementation of Disturbance Observer

We define auxiliary functions to avoid the realization of the estimated disturbance force through the acceleration of the microparticle

$$\Gamma = \hat{d} - \phi(\dot{y}), \quad (8)$$

where Γ and $\phi(\dot{y})$ are auxiliary functions (to be determined). The time derivative of Γ yields

$$\dot{\Gamma} = \dot{\hat{d}} - \frac{\partial \phi(\dot{y})}{\partial \dot{y}} \ddot{y}, \quad (9)$$

Since the disturbance observer is initially represented using a first-order low-pass filter as

$$\hat{d} = \frac{g}{s + g} (u - P_n^{-1}(s)\ddot{y}), \quad (10)$$

$$\dot{\hat{d}} + g\hat{d} = g (u - P_n^{-1}(s)\ddot{y}), \quad (11)$$

Implementation of Disturbance Observer

Substituting (8) and (9) in (11) yields

$$\dot{\Gamma} + \frac{\partial \phi(\dot{y})}{\partial \dot{y}} \ddot{y} + g\Gamma + g\phi(\dot{y}) = g(u - P_n^{-1}(s)\ddot{y}), \quad (12)$$

Setting the derivative of the auxiliary function, $\frac{\partial \phi(\dot{y})}{\partial \dot{y}} = -gP_n^{-1}(s)\ddot{y}$, yields the following representation of the disturbance observer:

$$\dot{\Gamma} = -g(\Gamma + \phi(\dot{y})) + gu \quad (13)$$

Finally the observer can be rewritten as follows:

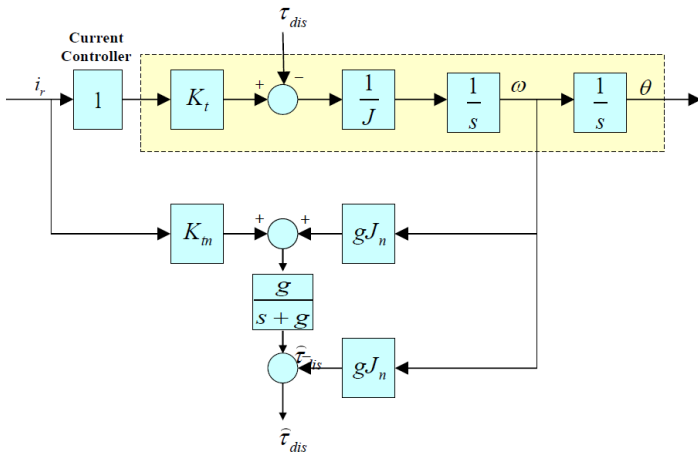
$$\hat{d} = \frac{g}{s + g} (u - \phi(\dot{y})) + \phi(\dot{y}). \quad (14)$$

$$\phi(\dot{y}) = -gP_n^{-1}(s)\dot{y} \quad (15)$$

Implementation of Disturbance Observer

$$\hat{d} = \frac{g}{s + g} (u - \phi(\dot{y})) + \phi(\dot{y}). \quad (16)$$

$$\phi(\dot{y}) = -gP_n^{-1}(s)\dot{y} \quad (17)$$



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