Assignment 2 - Deadline 1 week

Problem 1
A 4R planar robot with links of unitary lengths is shown in Fig. 1.

- Provide the Jacobian matrix $J(\theta)$ relating the joint velocity $\theta = \begin{pmatrix}\dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 & \dot{\theta}_1\end{pmatrix}^{\mathrm{T}}$ to the linear velocity $v = \begin{pmatrix}v_x & v_y\end{pmatrix}$ of the robot end-effector.
- Find all singular configurations of the Jacobian matrix $J(\theta)$.
- In the configuration $\theta = \begin{pmatrix}0 & 0 & \frac{-\pi}{4} & \frac{\pi}{2}\end{pmatrix}^{\mathrm{T}}$, determine the joint velocity $\dot{\theta}$ of minimum norm that realizes the desired end-effector velocity $v = \begin{pmatrix}1 & 0\end{pmatrix}$.

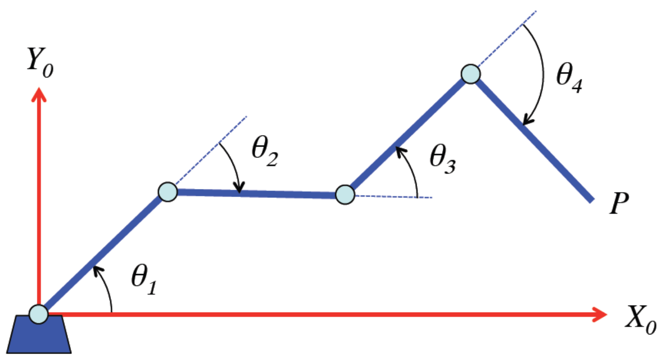

Figure 1: A planar 4R robot arm with unitary link lengths

Problem 2
Consider the orientation obtained through the sequence of three rotations specified by the angles α, β, and γ in Fig. 2. Pay attention to the definition of positive rotations. The figure shows a situation in which α and β have some positive values in $(0, \frac{\pi}{2})$.

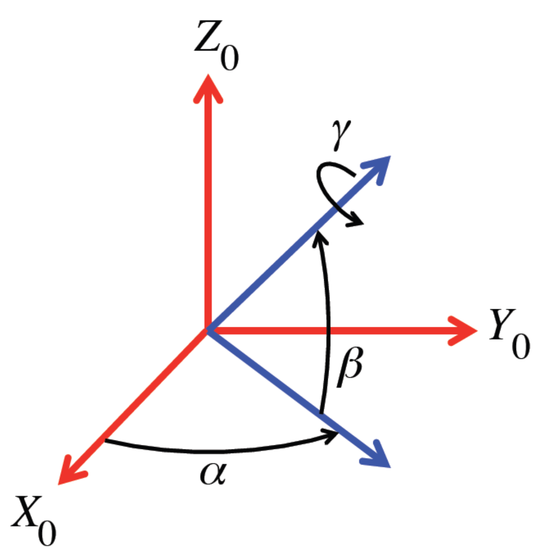

Figure 2: Definition of the three angles α, β, and γ

- Determine the associated rotation matrix $\mathbf{R}(\alpha, \beta, \gamma)$ (Forward kinematics problem).
- When the orientation is expressed by a rotation matrix $\mathbf{R}$, find the closed-form expressions for the minimal representation of orientation using the above set of angles α, β, and γ (inverse problem). Characterize the cases when two solutions or an infinite number of solutions exist.
- Obtain the mapping between the time derivatives of the three angles in this minimal representation and the angular velocity vector, i.e.,

$$\omega = \mathbf{T}(\alpha, \beta) \begin{pmatrix} \dot{\alpha} \\ \dot{\beta} \\ \dot{\gamma} \end{pmatrix} \tag{1}$$

  and find the singularities of the matrix $\mathbf{T}$. In one of these singularities, provide two numerical examples in which a desired ω can or, respectively, cannot be realized.