Robotics: Tutorial 6

Mechatronics Engineering

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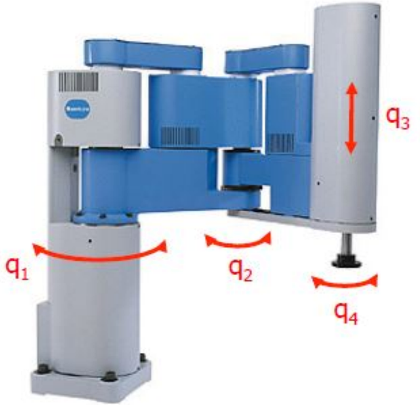
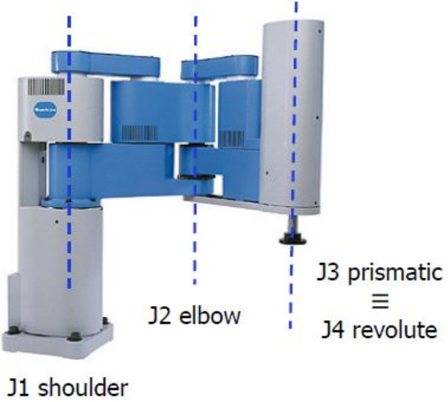
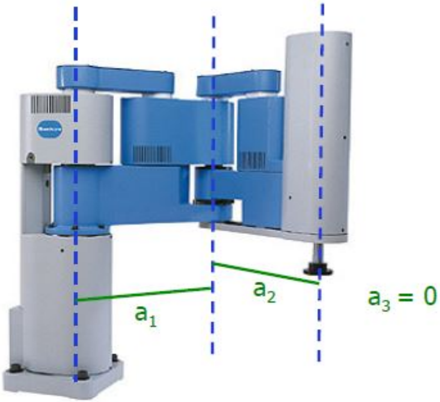
Question: Get the Kinematics of the following SCARA Robot using DH method.



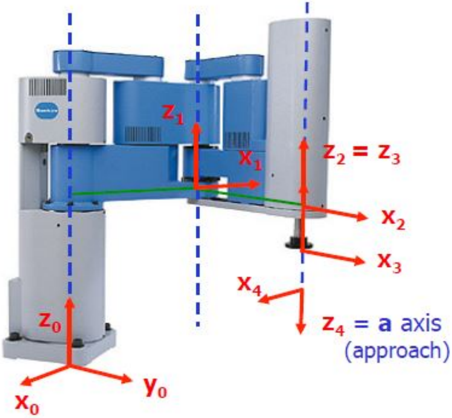
First Step: Label the joints and joint axes.



Second Step: Label the links and link axes



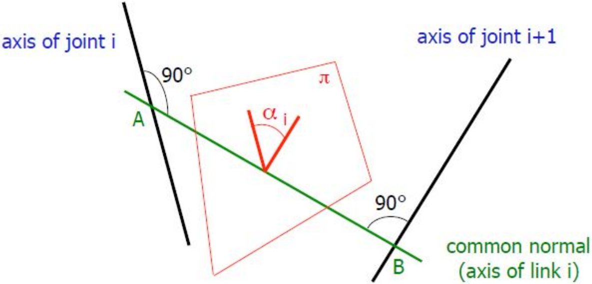
Third Step: Assign the frames.



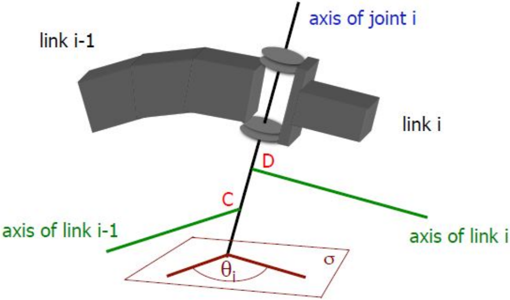
Rules for assigning the frames:

- Z axis is along the axis of rotation.
- Z_0 is assigned to J_1 and Z_1 is assigned to J_2 and so on, which means Z_n is assigned to J_{n+1}.
- X axis is assigned according to Z_n and Z_{n+1}, there is three cases:
  - if Z_n and Z_{n+1} are intersecting, then X-axis will be in the direction of their cross product.
  - if Z_n and Z_{n+1} are parallel to each other, then X-axis will be in the direction of the common normal.
  - if Z_n and Z_{n+1} coincide, then X_{n+1} will be in the direction of X_n.

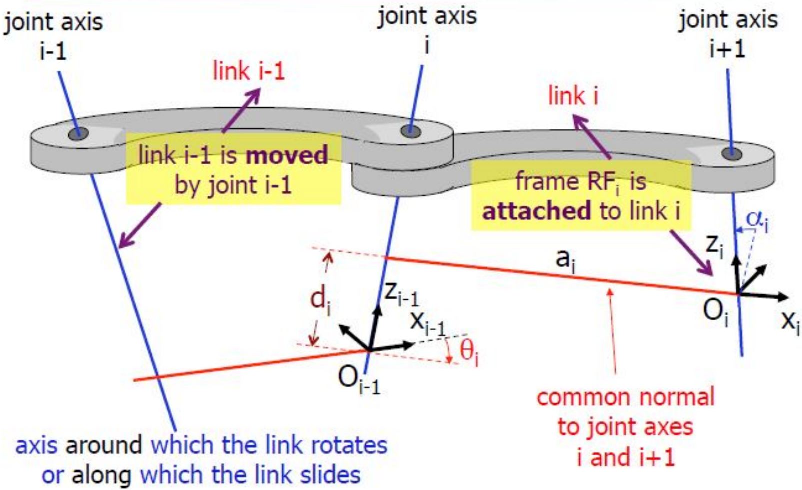
Forth Step: Forming the DH table of parameters:

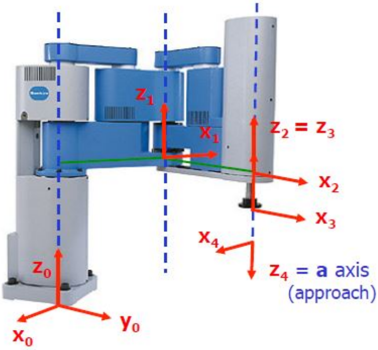


- a_i is the displacement between the joint axes(z-axes).
- α_i is the angle between the joint axes(z-axes).



- d_i is the displacement between the link axes(variable when joint is prismatic).
- θ_i is the angle between the link axes(variable when joint is revolute).


joint axis
i-1
joint axis
i
joint axis
i+1
link i-1
link i
link i-1 is moved
by joint i-1
frame RF$_i$ is
attached to link i
α_i
z_i
a_i
d_i
z_{i-1}
x_{i-1}
θ_i
O_{i-1}
O_i
x_i
axis around which the link rotates
or along which the link slides
common normal
to joint axes
i and i+1



i	α_i	a_i	d_i	θ_i
1	0	a_1	d_1	q_1
2	0	a_2	0	q_2
3	0	0	$-q_3$	0
4	π	0	$-d_4$	q_4

Fifth Step: Substitute by DH parameters in the DH matrix:

$$^{i-1}A_i(q_i) = \begin{bmatrix} cos\theta_i & -cos\alpha_i sin\theta_i & sin\alpha_i sin\theta_i & a_i cos\theta_i \\ sin\theta_i & cos\alpha_i cos\theta_i & -sin\alpha_i cos\theta_i & a_i sin\theta_i \\ 0 & sin\alpha_i & cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

$$
{}^{0}A_1(q_1) = \begin{bmatrix} cos(\theta_1) & -sin(\theta_1) & 0 & a_1 cos(\theta_1) \\ sin(\theta_1) & cos(\theta_1) & 0 & a_1 sin(\theta_1 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}
$$

$$
{}^{1}A_2(q_2) = \begin{bmatrix} cos(\theta_2) & -sin(\theta_2) & 0 & a_2 cos(\theta_2) \\ sin(\theta_2) & cos(\theta_2) & 0 & a_2 sin(\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}
$$

$$
{}^{2}A_3(q_3) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & q_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}
$$

$$
{}^{3}A_4(q_4) = \begin{bmatrix} cos(\theta_4) & sin(\theta_4) & 0 & 0 \\ sin(\theta_4) & -cos(\theta_4) & 0 & 0 \\ 0 & 0 & -1 & -d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}
$$

$$^{B}T_E = {}^{B}T_0 \, {}^{0}A_1 \, {}^{1}A_2 \, {}^{2}A_3 \, {}^{3}A_4 \, {}^{4}T_E = {}^{0}A_4 \tag{6}$$

Since ${}^{B}T_0 = {}^{4}T_E = I$

$$^{B}T_E = \begin{bmatrix} c\theta_1 c\theta_2 c\theta_4 & s\theta_1 s\theta_2 s\theta_4 & 0 & a_1 c\theta_1 + a_2 c\theta_1 c\theta_2 \\ s\theta_1 s\theta_2 s\theta_4 & -c\theta_1 c\theta_2 c\theta_4 & 0 & a_1 s\theta_1 + a_2 s\theta_1 s\theta_2 \\ 0 & 0 & -1 & d_1 - q_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

$$^{B}T_E = \begin{bmatrix} n & s & a & P_x \\ n & s & a & P_y \\ n & s & a & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

$$P_x = a_1 c\theta_1 + a_2 c\theta_1 c\theta_2, \tag{9}$$

$$P_y = a_1 s\theta_1 + a_2 s\theta_1 s\theta_2, \tag{10}$$

$$P_z = d_1 - q_3 - d_4 \tag{11}$$

Any Questions ?
Thank you