Robotics: Tutorial 8

Mechatronics Engineering

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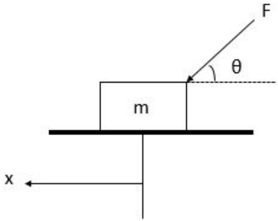
Principle of virtual work



$$Work = \underbrace{F cos\theta}_{\text{Force that contributes to the work}} \times displacement \quad (1)$$

Effect of a force on joint:

$$Work = F_i \frac{\partial |r^{oi}}{\partial q_i} \tag{2}$$

where,

$$F_i = \begin{bmatrix} F_{ix} & F_{iy} & F_{iz} \end{bmatrix} \tag{3}$$

And, $|r^{oi}$ is position vector from origin to point of application of force.

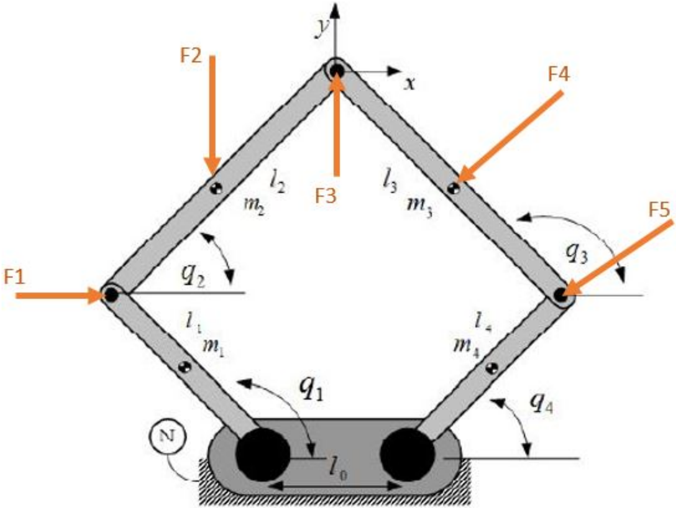
For joint (i):

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i \tag{4}$$

where,

$$Q_i = \underbrace{\tau_i}_{\text{in case of actuated joint}} + F_i \frac{\partial |r^{oi}}{\partial q_i} \tag{5}$$

For the shown Pantograph, study the dynamics of this robot under the effect of external forces.



Solution:
First, Get the dynamics by calculating KE and PE of four links to get Lagrangian equation (same method as in the previous tutorial)
Second, Use following equation for each joint:

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = \tau_i \quad (6)$$

Get the external forces:

$$F_1 : |r^{o1} = L_1 cos(q_1)|n_1 + L_1 sin(q_1)|n_2 \quad (7)$$

$$F_2 : |r^{o2} = (L_1 cos(q_1) + \frac{L_2}{2} cos(q_2))|n_1 + (L_1 sin(q_1) + \frac{L_2}{2} sin(q_2))|n_2 \quad (8)$$

$$F_3 : |r^{o3} = (L_1 cos(q_1) + L_2 cos(q_2))|n_1 + (L_1 sin(q_1) + L_2 sin(q_2))|n_2 \quad (9)$$

$$F_4 : |r^{o4} = (L_1 cos(q_1) + L_2 cos(q_2) - \frac{L_3}{2} cos(q_3)|n_1 + (L_1 sin(q_1) + L_2 sin(q_2) + \frac{L_3}{2} sin(q_3))|n_2 \quad (10)$$

$$F_5 : |r^{o5} = (L_1 cos(q_1) + L_2 cos(q_2) - L_3 cos(q_3)|n_1 + (L_1 sin(q_1) + L_2 sin(q_2) + L_3 sin(q_3))|n_2 \quad (11)$$

Therefore,

$$Q_1 = \tau_1 + F_1 \frac{\partial |r^{o1}}{\partial q_1} + F_2 \frac{\partial |r^{o2}}{\partial q_1} + F_3 \frac{\partial |r^{o3}}{\partial q_1} + F_4 \frac{\partial |r^{o4}}{\partial q_1} + F_5 \frac{\partial |r^{o5}}{\partial q_1} \quad (12)$$

$$Q_2 = \tau_2 + F_1 \frac{\partial |r^{o1}}{\partial q_2} + F_2 \frac{\partial |r^{o2}}{\partial q_2} + F_3 \frac{\partial |r^{o3}}{\partial q_2} + F_4 \frac{\partial |r^{o4}}{\partial q_2} + F_5 \frac{\partial |r^{o5}}{\partial q_2} \quad (13)$$

$$Q_3 = \tau_3 + F_1 \frac{\partial |r^{o1}}{\partial q_3} + F_2 \frac{\partial |r^{o2}}{\partial q_3} + F_3 \frac{\partial |r^{o3}}{\partial q_3} + F_4 \frac{\partial |r^{o4}}{\partial q_3} + F_5 \frac{\partial |r^{o5}}{\partial q_3} \quad (14)$$

$$Q_4 = \tau_4 + F_1 \frac{\partial |r^{o1}}{\partial q_4} + F_2 \frac{\partial |r^{o2}}{\partial q_4} + F_3 \frac{\partial |r^{o3}}{\partial q_4} + F_4 \frac{\partial |r^{o4}}{\partial q_4} + F_5 \frac{\partial |r^{o5}}{\partial q_4} \quad (15)$$

τ_2 and τ_3 will be equal to zero since they are both passive joints. Then Substitute by the force components and position vector differentiated with respect to q in Q_1 , Q_2 , Q_3 and Q_4.

$$Q_1 = \tau_1 + \begin{bmatrix} F_{1x} & F_{1y} & F_{1z} \end{bmatrix} \begin{bmatrix} -l_1 sin(q_1) \\ l_1 cos(q_1) \\ 0 \end{bmatrix}$$

$$+ \begin{bmatrix} F_{2x} & F_{2y} & F_{2z} \end{bmatrix} \begin{bmatrix} -l_1 sin(q_1) \\ l_1 cos(q_1) \\ 0 \end{bmatrix} + \begin{bmatrix} F_{3x} & F_{3y} & F_{3z} \end{bmatrix} \begin{bmatrix} -l_1 sin(q_1) \\ l_1 cos(q_1) \\ 0 \end{bmatrix}$$

$$+ \begin{bmatrix} F_{4x} & F_{4y} & F_{4z} \end{bmatrix} \begin{bmatrix} -l_1 sin(q_1) \\ l_1 cos(q_1) \\ 0 \end{bmatrix} + \begin{bmatrix} F_{5x} & F_{5y} & F_{5z} \end{bmatrix} \begin{bmatrix} -l_1 sin(q_1) \\ l_1 cos(q_1) \\ 0 \end{bmatrix}$$

Any Questions ?

Thank you